ECE 4880: RF Systems

Fall 2016

Chapter 1: Review of electromagnetic theory and resonator circuits

Reading Assignments:

1. Lectures 2, 3, 20, 21 and 22 of ECE 3030 notes
2. T. H. Lee, *The Design of CMOS Radio Frequency Integrated Circuits*, *2nd Ed*, Cambridge, 2004. Chap. 3

Logic Flow:

Topics for electromagnetic review:

- Maxwell equations in free space
- Guided waves and transmission lines
- Distributive waveguides and lumped-element discrete transmission line representations
- Smith Chart for reflection coefficients and impedance
- Stub lines for impedance matching

Topics for resonator review:

- LC resonator for impedance analysis
- Conjugate impedance match
- Quality factor and bandwidth of resonators

1.1 Maxwell equations in free space

We have been witnessing the wireless communication revolution in the last 30 years when smart phones have penetrated deep into the business and personal worlds. From the early days of satellite phones to today's Pokémon-Go, mobile personal units have changed lifestyles, if not life goals, for many people regardless of race, origin, and economical prosperity. In countries where the electrical power network has not yet set up, mobile network with solar-power base stations is often already available! The revolution has NOT finished though. With more automation from robots and social connectivity from Facebook and Uber, you will see mobile applications continue to bloom for the next 50 years.

Fig. 1.1. A hut in Africa for mobile phone connectivity

Do you wonder how "data" and "information" can come through the air? How can so many people share the "same free space" and information knows where to go? The magic lies in the electromagnetic (EM) wave propagation! Before we go any further, remember that EM waves do go to most places (including where you may not want them) and everyone shares the free space. Needless to say, regulation has to be set up for **safety** and **sharing**, not to mention many concerns for **security** and **privacy**!

We will start from the simplest model of EM wave propagation in air. For any wave propagation, we will look for the ratio of the second derivatives in time and space to be proportional with a constant v^2, where v is the propagation velocity. For example, for a 1D wave (3D can have more degrees of freedom) propagating in x with velocity v in the positive or negative x direction, the equation will look like:

$$\frac{\partial^2 a(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 a(x,t)}{\partial t^2} \qquad (1.1)$$

From the Maxell equations:

$$\begin{aligned}
&\nabla . \varepsilon_o \vec{E} = \rho \\
&\nabla . \mu_o \vec{H} = 0 \\
&\nabla \times \vec{E} = -\frac{\partial \mu_o \vec{H}}{\partial t} \qquad (1.2) \\
&\nabla \times \vec{H} = \vec{J} + \frac{\partial \varepsilon_o \vec{E}}{\partial t}
\end{aligned}$$

In free space, we have: $\rho = \vec{J} = 0$

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\nabla \times \left(\frac{\partial \mu_o \vec{H}}{\partial t}\right) = -\frac{\partial \mu_o \nabla \times \vec{H}}{\partial t} = -\mu_o \varepsilon_o \frac{\partial^2 \vec{E}}{\partial t^2}, \quad (1.3)$$

Defining: $c = \frac{1}{\sqrt{\varepsilon_o \mu_o}} \approx 3 \times 10^8$ m/s ,

$$\nabla \times \left(\nabla \times \vec{E}\right) = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} \qquad (1.4)$$

By using the vector identity: $\nabla \times \left(\nabla \times \vec{F}\right) = \nabla\left(\nabla \cdot \vec{F}\right) - \nabla^2 \vec{F}$, we can have the general 3D EM wave propagation equation as (in free space $\nabla \cdot \vec{E} = 0$)

$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}, \qquad (1.5)$$

or in scalar form of the electric field in the Cartesian coordinates:

$$(1)\quad \frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial y^2}+\frac{\partial^2 E_x}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}$$

$$(2)\quad \frac{\partial^2 E_y}{\partial x^2}+\frac{\partial^2 E_y}{\partial y^2}+\frac{\partial^2 E_y}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} \qquad (1.6)$$

$$(3)\quad \frac{\partial^2 E_z}{\partial x^2}+\frac{\partial^2 E_z}{\partial y^2}+\frac{\partial^2 E_z}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$$

1.2 Plane waves and the corresponding transmission line

If we now invoke the **plane wave**[1] assumption, i.e., $E_y = E_z = 0$ and

$$\nabla.\,\varepsilon_o\,\vec{E}=0 \qquad \Rightarrow \qquad \frac{\partial E_x}{\partial x}=0\text{, i.e., } E_x = E_x(z, t). \qquad (1.7)$$

One of the possible traveling wave solutions can be expressed as:

$$\vec{E}=\hat{x}\,E_x(z-ct)=\hat{x}\,E_o\cos\left(\frac{2\pi}{\lambda}(z-c\,t)\right)=\hat{x}\,E_o\cos(\omega t-k\,z) \qquad (1.8)$$

where we define the angular frequency $\omega=\frac{2\pi\,c}{\lambda}=2\pi\,f$ and wavevector $k=\frac{2\pi}{\lambda}$. We can derive the magnetic field as:

$$\frac{\partial \vec{H}}{\partial t}=-\frac{1}{\mu_o}\nabla\times\vec{E}=-\hat{y}\frac{k}{\mu_o}E_o\sin(\omega t-k\,z)$$

$$\Rightarrow \quad \vec{H}=\hat{y}\frac{k}{\mu_o\,\omega}E_o\cos(\omega t-k\,z) \qquad (1.9)$$

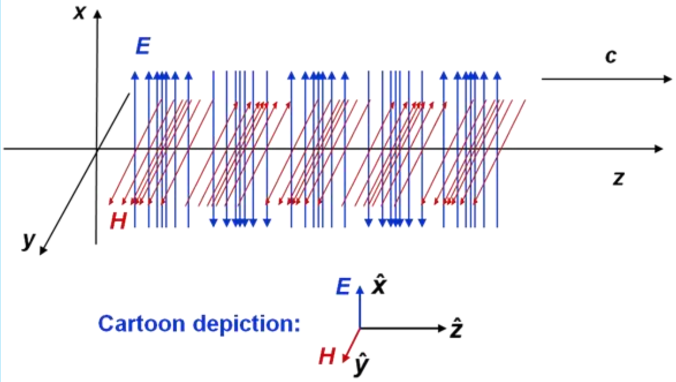
$$\Rightarrow \quad \vec{H}=\hat{y}\frac{E_o}{\eta_o}\cos(\omega t-k\,z)$$

η_o is the **impedance of the free space** at 377Ω. The plane wave with E in the x direction and H in the y direction, propagating in the z direction can be visualized in Fig. 1.2.

The result can be generalized for a 3D plane wave travelling in the $\vec{k}$ direction as:

$$\vec{E}(\vec{r},t)=\hat{n}\,E_o\cos(\omega t-\vec{k}\cdot\vec{r})\text{ where } \vec{k}\cdot\hat{n}=0. \qquad (1.10)$$

[1] Traveling plane waves in free space have the simplest form, but other waves are also commonly used, including the circular polarized waves.



Plane wave propagation

$$\vec{E} = \hat{x} E_o \cos(\omega t - k z)$$

$$\vec{H} = \hat{y} \frac{E_o}{\eta_o} \cos(\omega t - k z)$$

Fig. 1.2. The plane wave with *E* in the *x* direction and *H* in the *y* direction, propagating in the *z* direction.

We can make a few observations from the above derivation:

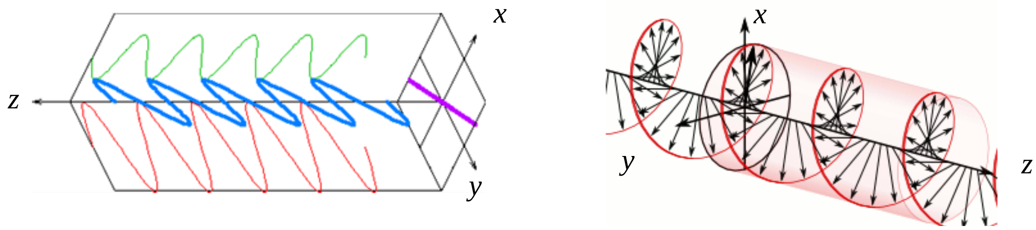
1. The plane wave solution is made by the assumptions of a specific functional form. Realistic solutions can have more complicated spatial dependence, although they can often be expressed as superposition of plane waves.

2. This solution is just from the propagating solution in free space with NO SOURCE (sources can be current in Hertzian dipole antenna, static charge, etc.) in sight. We did not include the source that generates this wave here at all. When the source has non-negligible size and we are observing within a quarter-wavelength, the solution can be very complicated. Electromagnetic coupling within a quarter-wavelength is termed as "**near field**", in contrast to the "**far field**" for plane wave representation. Near field communication **(NFC)** is still useful (such as card reading), but there is no propagating waves.

3. When we observe far away from the source, i.e., both the observation point (receiver antenna) and the "source" antenna are much smaller than the distance, planewave solutions can be representative.

4. The general 3D plane waves can have 2 degrees of freedom, as we only require: $\vec{k} . \hat{n} = 0$. This is part of the polarization. By convention, for a monopole receiving antenna, if the electric field is aligned with the antenna, it is called the TE (transverse electric) mode, and if the magnetic field is aligned with the antenna, it is called the TM (transverse magnetic) mode. We can have circular polarization as well, where the electric field direction has a periodic time dependence, as shown in Fig. 1.3.

5. The plane wave is also a solution for EM waves propagate in a metallic wave guide in the *z* direction, if the boundary condition fits, i.e., electric field is always normal to the metal surface. For example, a waveguide of metal plates in the *yz* plane will dictate that ONLY E_x is present, while $E_y = E_z = 0$, as no tangential electric field can exist on metal without causing a current. This will become a "transmission line".



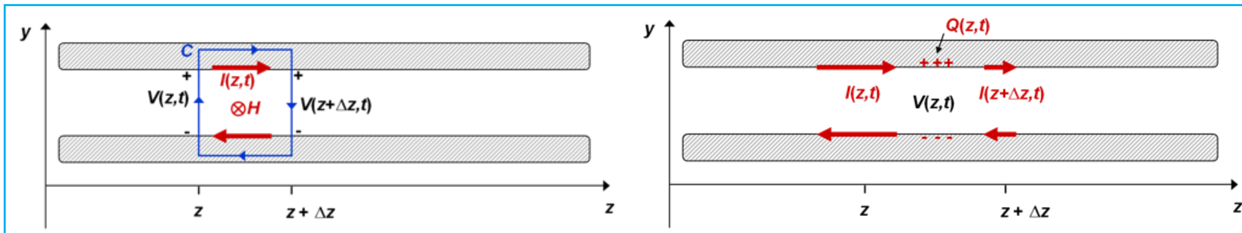
Linear polarization means the electric field oscillation can be projected into a line on the plane perpendicular to the traveling direction.

Circular polarization means the electric field oscillation can be projected into a circle on the plane perpendicular to the traveling direction.

Fig. 1.3. Linear and circular polarization of traveling waves.

1.3 Distributive and discrete transmission lines

In addition to free-space propagation, within a radio frequency (RF) unit, we often hope to guide the EM wave propagation so that the waves are confined in space, which will direct EM energy to the desirable receiving end and not disperse energy to unwanted places to cause interference. This is typically accomplished by a **transmission line** or a **waveguide**. We can visualize how the transmission line can be mapped into the 1D plane wave in free space, or in a 1D waveguide. A bit more cartoon[2] can help understand the mapping better. Notice that below the electric field $\vec{E}$ is in the *y* direction, and the magnetic field $\vec{H}$ is in the *x* direction.



$$\oint_c \vec{E}\,.\,d\vec{s} = -\frac{\partial}{\partial t}\iint \mu_o \vec{H}\,.\,d\vec{a}$$

$$\Rightarrow \quad V(z+\Delta z,t)-V(z,t) = -\frac{\partial LI(z,t)\Delta z}{\partial t}$$

$$\Rightarrow \quad \frac{\partial V(z,t)}{\partial z} = -L\frac{\partial I(z,t)}{\partial t}$$

$$I(z,t)-I(z+\Delta z,t) = \frac{\partial Q(z,t)\Delta z}{\partial t}$$

$$\Rightarrow \quad \frac{I(z+\Delta z,t)-I(z,t)}{\Delta z} = -\frac{\partial C\,V(z,t)}{\partial t}$$

$$\Rightarrow \quad \frac{\partial I(z,t)}{\partial z} = -C\frac{\partial V(z,t)}{\partial t}$$

Fig. 1.4. Voltage and current waves guided in transmission lines.

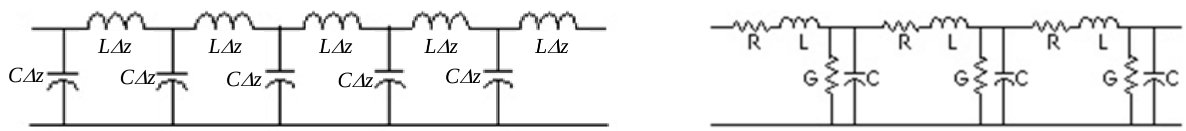
where *L* and *C* are the unit-length inductance and capacitance with units of H/cm and F/cm, respectively. The last two equations are called the Telegrapher's Equations

$$\frac{\partial V(z,t)}{\partial z} = -L\frac{\partial I(z,t)}{\partial t}; \qquad \frac{\partial I(z,t)}{\partial z} = -C\frac{\partial V(z,t)}{\partial t} \qquad (1.11)$$

[2] Visit the Wiki page: https://en.wikipedia.org/wiki/Transmission_line

We can also see that the discrete forms of the Telegrapher's Equations apply to the discrete lossless transmission line:

$$\Delta V(z,t) = -L\Delta z \frac{\partial I(z,t)}{\partial t}; \qquad \Delta I(z,t) = -C\Delta z \frac{\partial V(z,t)}{\partial t} \qquad (1.12)$$



A discrete lossless transmission line

A discrete lossy or dispersive transmission line

Fig. 1.5. Lossless and lossy discrete transmission lines by LC ladders.

L and *C* in the discrete transmission line above denote the inductance and capacitance per unit length in the distributed transmission line. When Δz is much smaller than the wavelength, the discrete transmission line behaves identically to the distributed transmission lines (or waveguides with the fundamental mode). The discrete lattice will behave differently when the segment Δz is "comparable" to the wavelength, or when the frequency is above the Bragg frequency of the LC lattice:

$$f_{bragg} = \frac{1}{\pi\sqrt{L\Delta z C\Delta z}} = \frac{1}{\pi\sqrt{LC}\Delta z} = \frac{v}{\pi\Delta z} \qquad (1.13)$$

Above f_{bragg}, the discrete transmission line cannot support a traveling-wave solution, and will only have evanescent wave solutions. Thus, the discrete lossless transmission line will behave like a low-pass filter with absolute (not just a 3dB corner frequency) cutoff frequency at f_{bragg}.

Let's work with the Telegrapher's Equations in the distributed transmission line. Most of the results are directly transferrable when the propagating wave has frequency below the cutoff frequency f_{bragg} of the LC lattice. We can easily write down the solution:

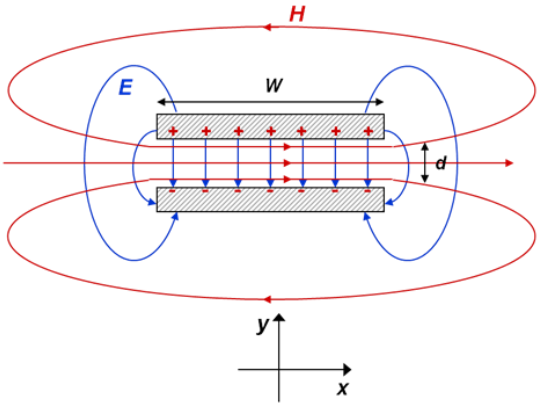
$$\frac{\partial^2 V(z,t)}{\partial z^2} = LC\frac{\partial^2 V(z,t)}{\partial t^2}; \qquad \frac{\partial^2 I(z,t)}{\partial z^2} = LC\frac{\partial^2 I(z,t)}{\partial t^2} \qquad (1.14)$$

That is, voltage and current travel down the transmission line with a velocity of $v = \frac{1}{\sqrt{LC}}$ in correspondence to the plane wave propagation in free space. The solution can be expressed in the phasor forms of travelling in the +*z* and –*z* directions:

$$\text{In } +z \text{ direction: } V(z) = V_+ e^{-jkz}; \qquad I(z) = I_+ e^{-jkz} = \frac{V_+}{Z_o} e^{-jkz} \qquad (1.15)$$

$$\text{In } -z \text{ direction: } V(z) = V_- e^{+jkz}; \qquad I(z) = I_- e^{+jkz} = -\frac{V_-}{Z_o} e^{+jkz} \qquad (1.16)$$

where $Z_o = \frac{\omega L}{k} = \sqrt{\frac{L}{C}}$ is the characteristic impedance of the transmission line. Let's reinforce with an example of a physical distributed transmission line and look at the solution from the transmission line and from the plane wave viewpoints. Popular transmission lines include coaxial cables, coplanar waveguides, and parallel-plate slot lines. Consider the parallel-plate slot lines (clearest for illustration but probably coaxial lines are most popular) with waves propagating in the *z* direction. The functional forms of the traveling waves of scalar *(V, I)* and vector $(\vec{E}, \vec{H})$ are shown in the figure below.



Transmission line variables:

$$\begin{cases} V(z) = V_+ \; e^{-jkz} \\ I(z) = I_+ \; e^{-jkz} = \dfrac{V_+}{Z_o} \; e^{-jkz} \end{cases}$$

Plane wave variables (ignoring fringing fields):

$$\begin{cases} \vec{E}(z) = -\hat{y} \; \dfrac{V_+}{d} \; e^{-jkz} \\ \vec{H}(z) = \hat{x} \; \dfrac{I_+}{W} \; e^{-jkz} \end{cases}$$

Fig. 1.6. Matchup of transmission-line and plane-wave waveforms from Maxwell phaser solutions.

This also serves as the example how the nodes in circuits are equi-potential planes in the Maxwell equation viewpoints.

The general transmission line solution is a superposition of the forward and backward waves:

$$V(z) = V_+ \, e^{-jkz} + V_- \, e^{+jkz}; \qquad (1.17)$$

$$I(z) = I_+ \, e^{-jkz} + I_- \, e^{+jkz} = \frac{V_+}{Z_o} e^{-jkz} - \frac{V_-}{Z_o} e^{+jkz} \qquad (1.18)$$

The distributive and discrete transmission lines above are "lossless". When loss is considered as wire resistance and dielectric loss, it will appear as an additional "diffusion" term in the wave equation (second order in space and first order in time). A sinusoidal travelling wave will become attenuated (energy loss) and change in waveforms (dispersion). Luckily, a copper coaxial RF cable often has very small loss, typically < 1 dB per 100 m.

Exercise:

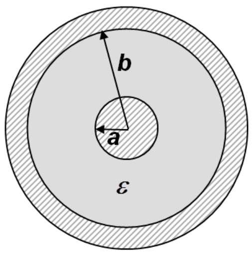
For the parallel-plate slot line above with width W and gap d, what will be the unit-length *L* and *C* to model it as a transmission line?

Solution: $C = \varepsilon \frac{W}{d}$; $\quad L = \mu_0 \frac{d}{W}$; $\quad v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu_0 \varepsilon}}$.

Notice that the electromagnetic waves are really in the dielectric, so the propagation velocity is determined by the dielectric constant at the given frequency (slower than speed of light).

Exercise:

For a co-axial cable as shown below, how should we define the unit-length *L* and *C* to model it as a transmission line? Illustrate how $(\vec{E},\vec{H})$ will look like in the *xy* plane with the electromagnetic waves propagate in the *z* direction (out of the paper).



Solution:

$$C = \frac{2\pi\,\varepsilon}{\log\left(\frac{b}{a}\right)}$$

$$L = \frac{\mu_o}{2\pi}\log\left(\frac{b}{a}\right)$$

$$LC = \mu_o\,\varepsilon$$

Fig. 1.7. Coaxial cables in the discrete transmission-line representation.

A closing remark on the transmission line: both the distributed and lumped transmission lines are linear circuits or linear elements, i.e., a monotone excitation will only have monotone output in the same frequency, even when we include resistive and linear dielectric loss. However, this does not mean the waves with combination of frequency or wavepacket will travel without distortion or dispersion (dispersion is defined when the phase velocity depends on frequency). Consider two waves $A_1\cos(\omega_1 t - k_1 z) + A_2\cos(\omega_2 t - k_2 z)$ in the transmission line. If A_1 and A_2 decay differently by resistive loss, the wave composition will change. Furthermore, if the phase velocity $d\omega/dk$ depends on either *A* (through nonlinear capacitance) or ω (through frequency-dependent perimitivity), then the wave composition observed at a given *t* can be further distorted. If the wave composition or shape, interpreted as the ratio of each frequency component (only A_2/A_1 here), does not change during propagation when the effect of loss is cancelled by nonlinearity in *A* or ω, we called this propagation "soliton" or "self-reinforcing solitary wave. Soliton circuits and waveguides are however NOT used in typical radios (used more often in equipment), and will not be covered in this class.

1.4 Transmission Line Analysis: General

The general transmission line solution in the AC expression is a superposition of the forward and backward waves (time-domain transient will come later):

$$V(z) = V_+\,e^{-jkz} + V_-\,e^{+jkz}; \qquad (1.19)$$

$$I(z) = I_+\,e^{-jkz} + I_-\,e^{+jkz} = \frac{V_+}{Z_o}e^{-jkz} - \frac{V_-}{Z_o}e^{+jkz} \qquad (1.20)$$

where $Z_0 = \sqrt{\frac{L}{C}}$; $k = \omega\sqrt{LC} = \frac{\omega}{v} = \frac{2\pi}{\lambda}$.

Most common transmission lines in RF systems:

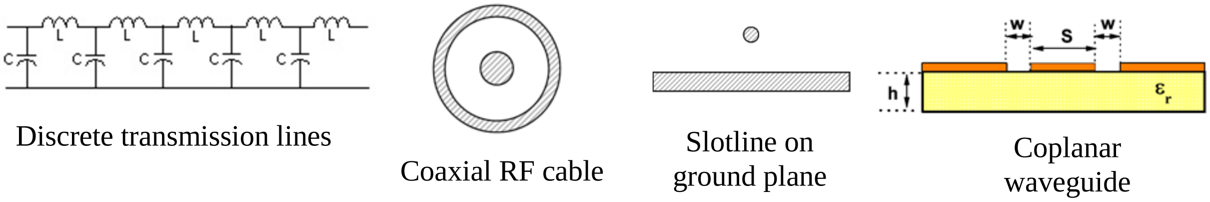


Fig. 1.8. Common implementation of transmission lines and waveguides.

Transmission line circuits can be readily understood from the source and load connections:

- Z_0: transmission line impedance
- Z_L: load impedance (can be complex number)
- Z_S: source impedance (can be complex number)

We will denote the load end as $z = 0$, and source end as $z = -\ell$ to describe the traveling waves of V_+ and V_- on the transmission line.

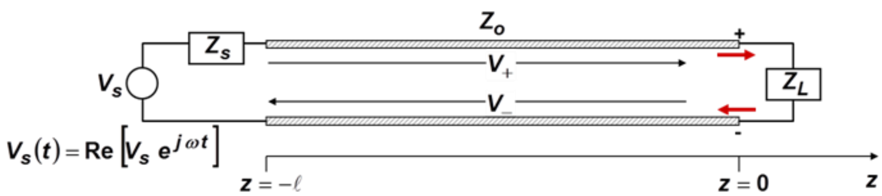


Fig. 1.9. A distributive (continous) transmission line with source drivers (V_S and Z_S) and termination load (Z_L).

At the load end ($z = 0$), we can write the termination of the two traveling waves as:

$$\frac{V(z=0)}{I(z=0)} = \frac{V_+ + V_-}{V_+/Z_o - V_-/Z_o} = Z_L \; ; \Rightarrow \quad \frac{V_-}{V_+} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} \qquad (1.21)$$

We can thus define the **load reflection coefficient** Γ_L at the load as:

$$\Gamma_L = \frac{V_-}{V_+} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} \qquad (1.22)$$

<u>1.5 Open, Short, Matched and General Loads</u>

For short-circuit load (i.e., $V(z = 0) = 0$):

$$\Gamma_L = \frac{V_-}{V_+} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} = -1 \qquad \Rightarrow \qquad V_- = -V_+ \qquad (1.23)$$

$$V(z=0) = V_+ + V_- = 0 \text{ and } I(z=0) = \frac{V_+}{Z_o} - \frac{V_-}{Z_o} = 2\frac{V_+}{Z_o}. \qquad (1.24)$$

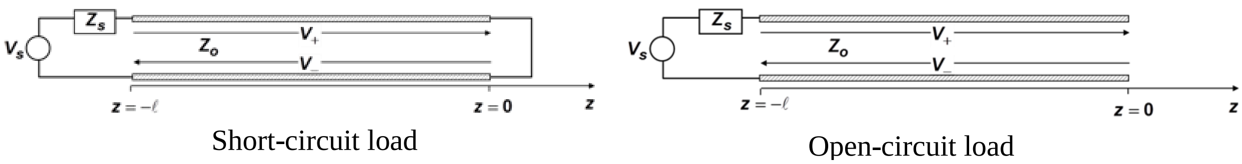


Fig. 1.10. Traveling waveforms V_+ and V_- encountering short-circuit and open-circuit loads.

For open-circuit load (i.e., $I(z = 0) = 0$):

$$\Gamma_L = \frac{V_-}{V_+} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1} = +1 \qquad \Rightarrow \qquad V_- = V_+ \qquad (1.25)$$

$$V(z=0) = V_+ + V_- = 2V_+ \text{ and } I(z=0) = \frac{V_+}{Z_o} - \frac{V_-}{Z_o} = 0 \qquad (1.26)$$

If we observe the traveling wave at an arbitrary position z, then we can express the forward and reverse traveling waves by using the reflection coefficient Γ_L:

$$V(z) = V_+\left(e^{-jkz} + \Gamma_L\, e^{+jkz}\right); \qquad I(z) = \frac{V_+}{Z_o}\left(e^{-jkz} - \Gamma_L\, e^{+jkz}\right) \quad (1.27)$$

If we observe the impedance at position z, the impedance towards the load will be:

$$Z(z) = \frac{V(z)}{I(z)} = Z_o \frac{1+\Gamma_L\, e^{2jkz}}{1-\Gamma_L\, e^{2jkz}}, \qquad (1.28)$$

which contains a real part of resistance and an imaginary part of reactance (okay, Smith Chart is almost there)…

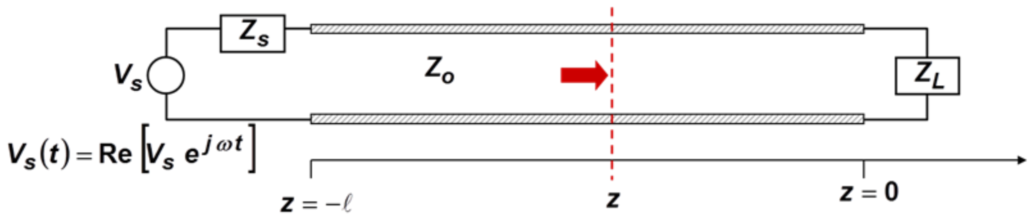


Fig. 1.11. Transmission-line impedance solution at a given line position *Z(z)*.

Check: $Z(z=0) = Z_o \left.\dfrac{1+\Gamma_L\, e^{2jkz}}{1-\Gamma_L\, e^{2jkz}}\right|_{z=0} = Z_o \dfrac{1+\Gamma_L}{1-\Gamma_L} = Z_L$

1.6 Observation of Equivalent Circuits at the Source End

At the source end, if we look towards the load, we can observe an impedance of $Z(z=-\ell)$:

$$Z(z=-\ell) = \frac{V(z=-\ell)}{I(z=-\ell)} = Z_o \left.\frac{1+\Gamma_L\, e^{2jkz}}{1-\Gamma_L\, e^{2jkz}}\right|_{z=-\ell} = Z_o \frac{1+\Gamma_L\, e^{-2jk\ell}}{1-\Gamma_L\, e^{-2jk\ell}} \qquad (1.29)$$

Equation (1.29) is similar to, but more complex and flexible than, the resonator impedance transfer circuits by changing the length of the transmission line. We can write Eq. (1.29) to find the equivalent view of the reflection coefficient:

$$\Gamma(z) = \Gamma_L e^{2jkz} = \frac{Z(z)-Z_0}{Z(z)+Z_0} = \frac{Z(z)/Z_0 - 1}{Z(z)/Z_0 + 1} \qquad (1.30)$$

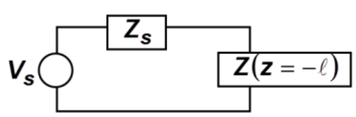


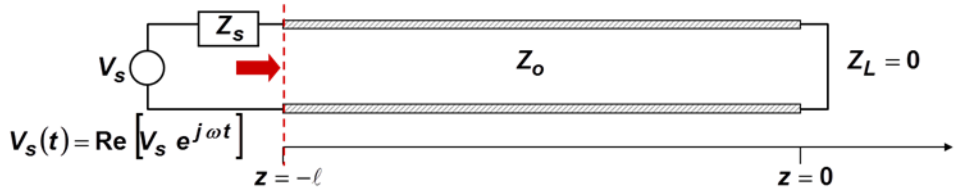
Fig. 1.12. Transmission-line impedance at the source end. Notice that $Z(z=-\ell)$ is a function of Z_L and ℓ.

We can further solve the above source-load circuit as:

$$V(z=-\ell) = V_+\left(e^{jk\ell} + \Gamma_L\, e^{-jk\ell}\right) = V_S \frac{Z(z=-\ell)}{Z_S + Z(z=-\ell)} \qquad (1.31)$$

The degree of freedom is simply ℓ !!! In the old days, this is a variable-length transmission line!!! Let's look at several asymptotic cases.

Consider a short circuit load, we can have:



$$Z(z=-\ell) = Z_o \frac{1+\Gamma_L\, e^{-2jk\ell}}{1-\Gamma_L\, e^{-2jk\ell}} = Z_o\ j\tan(k\,\ell)$$

Fig. 1.13. Transmission-line impedance at the source end with short-circuit termination. Notice that the impedance at $z=-\ell$ is a function of ℓ only.

At low frequency, no waves really travelling as $\omega << \frac{v}{\ell}; \ell << \frac{\lambda}{2\pi}; k\ell << 1$, we have:

$$Z(z=-\ell) = Z_o\ j\tan(k\,\ell) \approx j\,Z_o\,k\ell = j\sqrt{\frac{L}{C}}\,\omega\sqrt{LC}\,\ell = j\,\omega\,(L\ell) \qquad (1.32)$$

That is, the transmission line is like ONE inductor. It should, the discrete transmission line applies here, and the shunt capacitance is shorted out. Or you can simply see this as a one-turn solenoid! You can probably guess that for open load at low frequency, the transmission line will be like ONE capacitor. The transmission line equation goes like:

$$Z(z=-\ell) = -Z_o\ j\cot(k\,\ell) \approx -j\,\frac{Z_o}{k\ell} = -j\sqrt{\frac{L}{C}}\,\frac{1}{\omega\sqrt{LC}\,\ell} = \frac{1}{j\,\omega(C\ell)} \qquad (1.33)$$

Yes, as the two lines are separated by a dielectric, it will be a capacitor!

One more interesting case is the match load, where $\Gamma_L = 0$. The impedance at any point in z is Z_0!!!

1.7 Periodicity and SWR

Now we will take a look at the general Z_L case and make a few more observations. As previously, the impedance at any z can be expressed as:

$$Z(z) = Z_o \frac{1+\Gamma_L\, e^{2jkz}}{1-\Gamma_L\, e^{2jkz}} \qquad (1.34)$$

We can notice that the periodic behavior of the impedance:

$$Z(z) = Z(z + m\pi/k) = Z(z \pm m\lambda/2). \qquad (1.35)$$

That is, the impedance at z will repeat itself for every half wavelength.

Finally, if the forward and reverse-going waves interfere with each other, a standing wave can be formed in the steady state. Recall: $\Gamma_L = \frac{V_-}{V_+} = \frac{Z_L/Z_o - 1}{Z_L/Z_o + 1}$, and Z_L can be a complex number.

$$V(z) = V_+\, e^{-jkz} + V_-\, e^{+jkz} = V_+ \left(e^{-jkz} + \Gamma_L\, e^{+jkz}\right) \text{ with } \Gamma_L = |\Gamma_L|\, e^{j\phi} \qquad (1.36)$$

We can see the magnitude of the standing wave will be:

$$\begin{aligned} &\Rightarrow \quad |V(z)|^2 = |V_+|^2 \left\lfloor 1 + |\Gamma_L|^2 + 2\,|\Gamma_L| \cos(2kz + \phi) \right\rfloor \\ &\Rightarrow \quad |V(z)| = |V_+| \sqrt{1 + |\Gamma_L|^2 + 2\,|\Gamma_L| \cos(2kz + \phi)} \end{aligned} \qquad (1.37)$$

We can define a standing wave ratio (SWR):

$$SWR = \frac{|V(z)|_{\max}}{|V(z)|_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \qquad (1.38)$$

Z_0, Z_L, Γ_L and SWR are different ways to look at a transmission line of Z_0 terminated with Z_L. They are not independent to each other, and are used interchangeably most often due to convenience of viewing or measuring. Simultaneous representations of all parameters can be viewed in the Smith Chart (next lecture).

As a closing remark on the RF system in general, radio components (filters, amplifiers, mixers, etc.) are made individually (to be of general-purpose components) with unavoidable variations. When a radio engineer (especially in the old days without RFIC) put together a transceiver, each component is measured to make sure their role in the system has everything needed. Just like good carpentry: "Measure twice, cut once"!!! Once you deliver the whole radio system, many times it will be too late to know the component variations. Why do we need such perfection? In a transceiver, as the power we need to deal with has a huge range (say 1W transmission and 1pW reception), we need to know the small details, because a tiny reflection of the transmitter can possibly totally pollute your receiving signal chain!

Exercise:

A step pulse (very seldom in the RF signal, but common in digital or baseband) is sent on a transmission line with $Z_S = Z_0$. If the receiving end can be approximated with an open circuit (a small capacitance), describe the transient V_+, V_- and ($V_+ + V_-$) waveforms. Notice that you cannot use the traveling wave here, as the step pulse contains many frequencies. However, the impedance and reflection coefficient in the Thevenin circuits remain valid.

1.8 Transmission line analysis by Smith Charts

Smith Chart is invented by Phillips Smith in 1930s as a visualization tool for the impedance *Z* or admittance *Y* with respect to the reflection coefficient Γ of the transmission line, or in general, a signal chain connected by the transmission lines. It is also useful to guide the antenna design for impedance matching. Actually it can be applied to any system solution that deals with the superposition of forward and backward going waves (like in oscillators or optics). Extended Smith Chart can be used for noise figures, gain contours and stability analysis. In 2015, all copyright was now transferred to IEEE in 2015!

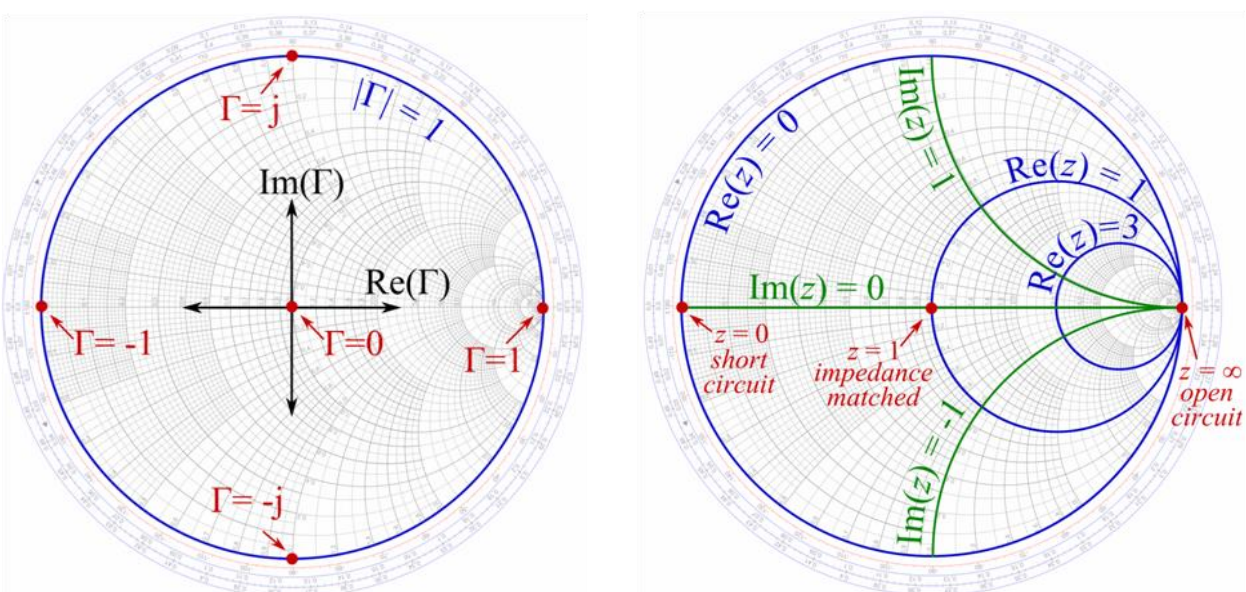
Mathematically, Smith Chart is plotting the contours of the real (west-most point is 0 and east most point is ∞ in circles) and imaginary (0 at west-east line, north-most point is 1 and south-most point is -1 in radials) parts of a quantity Z_n on the polar plane of Γ (|Γ| is the radius and ∠Γ is the azimuth angle), where Z_n and Γ are related by:

$$Z_n = \frac{1+\Gamma}{1-\Gamma} \qquad \text{or} \qquad \Gamma = \frac{Z_n - 1}{Z_n + 1} \qquad (1.39)$$

The design parameters will be most useful when it is linearly dependent on ∠Γ, such as the length parameter *z* in the transmission line. Most analyses can then be dealt with by a ruler and a compass. As most engineers are lazy, usually a ruler is good enough.

Surely, computers will not need the Smith Chart, but it can help you simultaneous view Z_n and Γ (and other variables in the future). We should first observe for any Z_n, we would have $|\Gamma| \le 1$ (to be in the polar coordinates). In any given point in the Smith Chart, we can simultaneously read four quantities from the labels on the chart: |Γ|, ∠Γ, Re(Z_n) and Im(Z_n).

The east-most point will need some explanation, as it can represent any real part of Z_n. We will look at this degeneracy to cast away future doubt when we land at this point. Notice that the west-most point of $\Gamma = -1$ can only be true if $Z_n = 0$. The most obvious solution of $\Gamma = 1$ is $Z_n \to \infty$, which corresponds to many possible scenarios, as either the Re(Z_n) OR Im(Z_n) approaches ∞ will be sufficient.



Γ as a complex number in the polar coordinates.

Contours of real and imaginary parts of $z = Z_n$ plotting on the *Γ* polar coordinates.

Fig. 1.14. Use of the Smith chart to find the reflection coefficient and complex impedance.

Let's see how the Smith Chart can be applied to the convenient use of the transmission line analysis. Remember that all analyses below are based on fixed *k*, *λ* and *ω*, i.e., valid only in a very narrow band.

From the previous lecture, we know that the impedance of the transmission line with a general load can be expressed as:

$$Z(z) = \frac{V(z)}{I(z)} = Z_o \frac{1+\Gamma_L e^{2jkz}}{1-\Gamma_L e^{2jkz}}, \qquad (1.40)$$

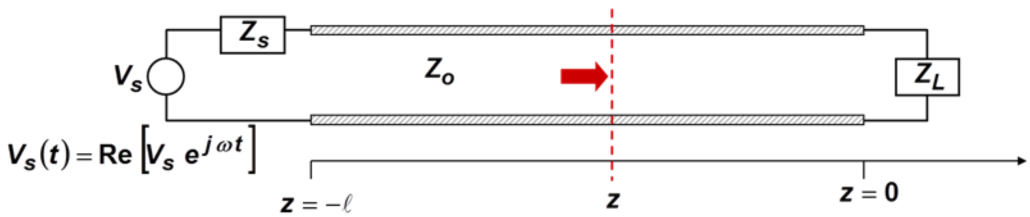


Fig. 1.15. The transmission-line impedance at arbitrary position between $(-\ell, 0)$.

If we define a reflection coefficient (as a complex number) anywhere at location *z* as:

$$\Gamma(z) = \Gamma_L e^{2jkz} \qquad (1.41)$$

We then have the relation between $Z_n(z) = Z(z)/Z_0$ and $\Gamma(z)$ as:

$$Z_n(z) = \frac{1+\Gamma(z)}{1-\Gamma(z)} \quad \text{and} \quad \Gamma(z) = \frac{Z_n(z)-1}{Z_n(z)+1} \qquad (1.42)$$

We can see this corresponds to the mathematical description in the Smith Chart. The change in *z*, from observing $\Gamma(z) = \Gamma_L e^{2jkz}$, is represented by the $2kz = \angle\Gamma$, where $k = \omega\sqrt{LC} = \frac{\omega}{v} = \frac{2\pi}{\lambda}$. So, if we know Γ_L at $z = 0$ (we know $|\Gamma_L| \leq 1$) for one point in the Smith Chart, $\Gamma(z) = \Gamma_L e^{2jkz}$ will be the contour of a center circle when *z* goes to $-\lambda/2$. Then both Γ(z) and Zn(z) will periodically change for every $\lambda/2$.

In the figure below if we have $Z_0 = 50\Omega$, $Z_L/Z_0 = Z_n(z=0) = 1.0 + j1.4$ (i.e., 50Ω resistance and 70Ω positive/inductive reactance), we can use that point to draw a circle at center in the clockwise rotation, which represents how Z_n changes when *z* moves from 0 to $-\lambda/2$. We can easily identify how long the transmission line needs to be when *Z* is real by the Smith Chart. The contour passes the X-axis (where Im(Z) = 0) two times, one at $Z_n = 4$; $Z = 200\Omega$, and the other at $Z_n = 0.3$; $Z = 15\Omega$. We can also see what *Z* can be possibly achieved from the circle.

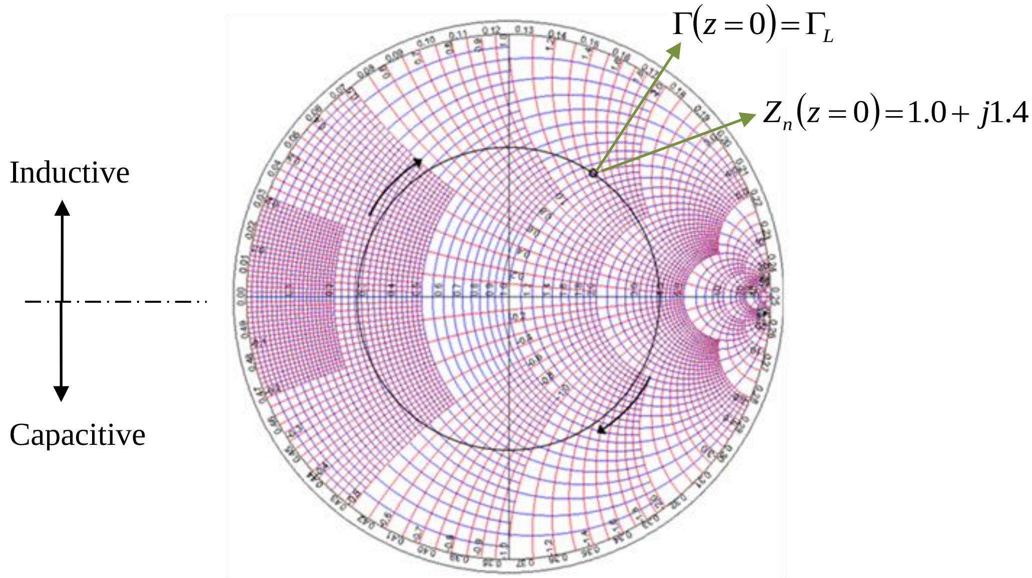


Fig. 1.16. The upper hemisphere of the Smith Chart has positive imaginary part for impedance and hence inductive, while the lower hemisphere is capacitive.

One more practice before we will introduce the Stub tuning and quarter-wave matching. Suppose, we start with a load of 10Ω, and hope to match the real part to 50Ω at the source end (do not care the imaginary part yet, as you can see, you cannot work on both just using one transmission line).

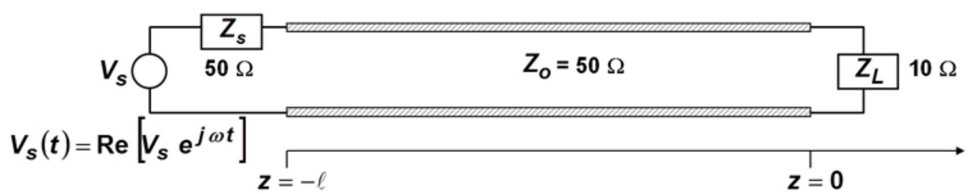


Fig. 1.17. A calculation example of a transmission line with characteristic impedance of 50 Ω.

We first know that Z_n = 10/50 = 0.2 and $\Gamma_L = \dfrac{Z_L/Z_o - 1}{Z_L/Z_o + 1} = -\dfrac{2}{3} = -0.667$ (in the complete Smith Chart, -0.667 can be read from the bottom). We can then construct the center circle and see that it will intercept the Re(Z_n) = 1.0 circle two times. We will take one in the upper hemisphere, which will be 50Ω plus an inductance, and we can read 0.18λ from the outer perimeter.

$$Z_n(z = -0.18\lambda) = (1.0 + j1.8);\ Z(z = -0.18\lambda) = 50 + j90.$$

<u>1.9 Conjugate matching by series and parallel stub tuners</u>

To cancel the imaginary part after matching the real part of Z, we can use the series stub tuners:

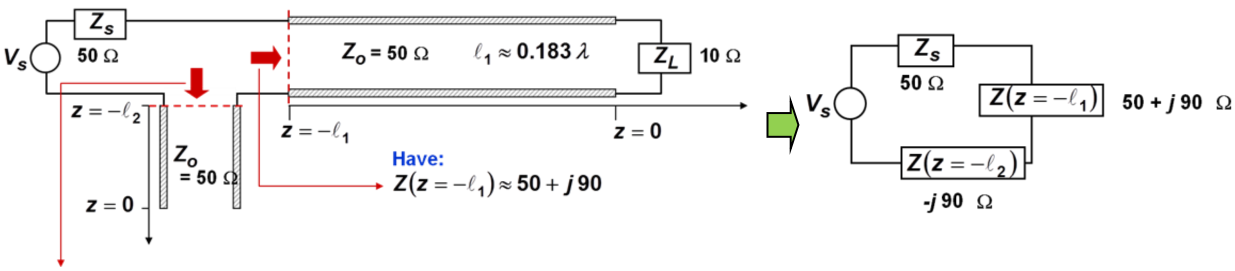


Fig. 1.18. A open-circuit stub tuner in series of a transistor with a realistic load for canceling the imaginary part of the impedance at the source end (to achieve the conjugate matching condition).

We can see we will need a series of −j90 (capacitive) to cancel the inductive reactance. This can be accomplished by a Z_0 = 50Ω transmission line with an open load (most convenient) and length to be found in the Smith Chart. Starting from the open circuit at the east-most point, we can find −1.8 in the lower hemisphere on the outmost circle with a wavelength number of 0.33, where we can find the length of the stub should be: $0.33\lambda - 0.25\lambda = 0.08\lambda$.

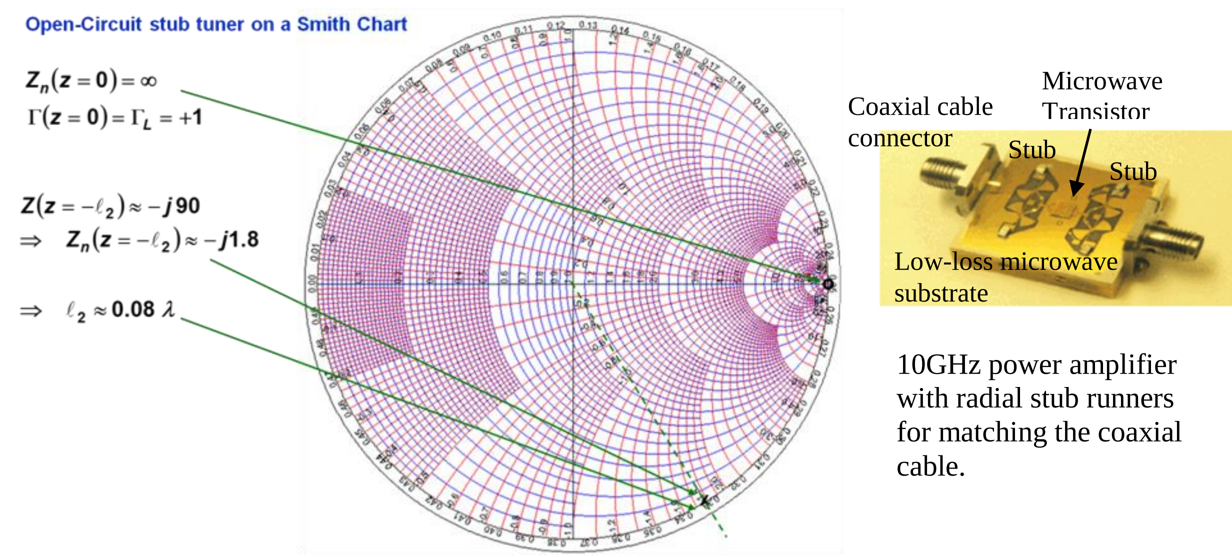


Fig. 1.19. Example and photo of stub tuners in microwave amplifiers.

You can use a parallel stub runner as well. We know that admittance is more convenient to use in parallel network. Luckily, admittance can be read from the Smith Chart readily with 180º phase shift, because

$$Y(z)=\frac{1}{Z(z)};\qquad Y_n(z)=\frac{Y(z)}{Y_o}=\frac{1}{Z_n(z)}=\frac{1-\Gamma(z)}{1+\Gamma(z)}\qquad (1.43)$$

Stub runners are used extensively in microwave circuits, and are useful to the antenna matching in the radio design as well.

Notice that all analyses done previously assume fixed frequency and wavelength, and is ONLY valid within a small spectrum of the carrier frequency.

Exercise:

If the transmission line has quarter wavelength, how does it transform Z_L to the source end?

Answer: quarter wavelength is half circle, so it will just be the inverse (notice that Smith Chart works in normalized numbers). Now $Z_1 = Z(z = -\lambda/4) = Z_0(1/Z_n) = Z_0^2/Z_L$. This is actually one of the easiest ways to obtain Z_0 by measurements (as infinite lines do not exist, and you cannot easily match it without knowing the exact Z_0).

If the transmission line has half wavelength, then we will have Z_1?

1.10 RLC resonators and resonance frequency

Maxwell's equations are the physical truth describing the geometrical aspects of the electric and magnetic field profiles, subject to the boundary conditions and sources (net charge, current and magnetic dipole). Circuits, as in simulation program with an integrated circuit emphasis (SPICE), are topological descriptions of the voltage and current of the circuit nodes (integration of the equi-potential planes as the Maxwell's equation solution, which will become clearer in the plane wave propagation example).

R, *L* and *C* are 0D, 2-terminal circuit elements, when the physical implementation can be approximated by the topological description of two circuit nodes:

$V = ZR$ or

$v = iR$	
$v = L\dfrac{di}{dt} = j\omega L \cdot i$	$Z_L = j\omega L$
$i = C\dfrac{dv}{dt} = j\omega C \cdot v$	$Z_C = \dfrac{1}{j\omega C}$

Passive elements are very often used in RF design for filtering, impedance match and resonating circuits. They are often referred as RLC network or RLC banks. In RF circuits, we used both lumped elements like RLC and transistors, as well as distributed elements such as antenna, to accomplish radiation of the selective electromagnetic waves into the space.

Consider an example of RLC resonator (parallel network can be thought for current amplification in a selected frequency, or the resonance frequency):

$$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = \frac{1}{Z_{in}} \qquad (1.44)$$

Y and Z_{in} become real at the angular frequency: $\omega_o = \dfrac{1}{\sqrt{LC}}$

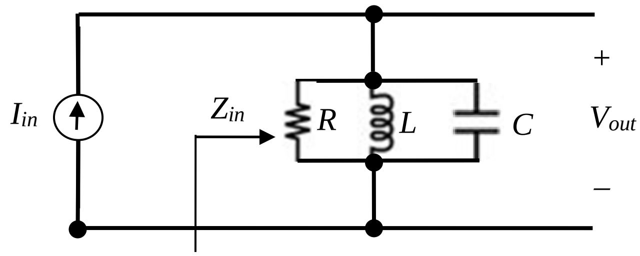


Fig. 1.20. An RF resonator with RLC in parallel.

For fast estimation, 1nH and 1pF give 5GHz resonating frequency. The radio frequency of interest today is in the range of 500MHz to about 10GHz (some extended to 40GHz, but not much more), which is in good correspondence of what can be easily achieved on PCB, or in many IC technologies (although the passive elements of nH and pF will look a lot larger than the transistors). 500MHz is often constrained by the size of the antenna, as efficient antenna needs to be "about" or larger than the quarter wavelength. 1GHz has wavelength in air for 30cm, and the quarter wavelength at 7.5cm is about your palm size (not counting fingers) that can be easily held. With bandwidth as a percentage of the carrier frequency, 500MHz is also constrained from the data rate. For example, if a channel occupies 0.5% of the carrier frequency at 1GHz, the bit rate is around 5 Mb/s, which is be larger or smaller by the SNR and modulation methods. The upper bound of 40 GHz is often limited by the free-space loss, which has two major factors: the range given by Frii's far-field space loss is proportional to λ^2 (which will be clear in the future) and the propagation attenuation factor α (in cm^{-1}) as the absorption coefficient in air increases with frequency until we reach close to the spectrum of visible light.

Far-field transmission with ONLY Frii's free-space loss (this is how the radiation energy is spread on the propagating spheres):

$$P_{receiver} = P_{source}\left(\frac{\lambda}{4\pi r}\right)^2 \Psi_s \Psi_r \qquad (1.45)$$

where $P_{receiver}$ is the power at the receiver antenna, P_{source} is the power transmitted at the source antenna, and Ψ_s and Ψ_r are the antenna gains of the source transmitter and the receiver.

The absorption coefficient in air (as a function of the air content, especially moisture) is typically:

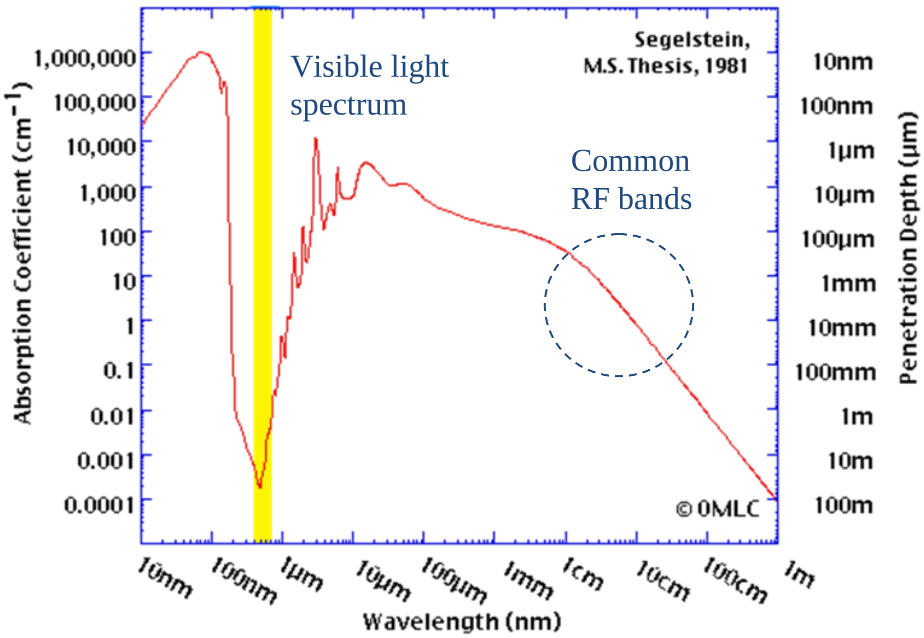


Fig. 1.21. The absorption coefficient (imaginary part of the dielectric constant) in the EM spectrum.

Important observation: At ω_o, the impedances of *L* and *C* cancel each other, but the transient or AC current in *L* and *C* can become VERY large, which is the essence of resonance. The energy is stored in the system and oscillating between *L* and *C*, and it looks from the outside the IV relationship can be described by *R* only.

1.11 The meaning and use of *Q*

How huge can be oscillating waves in the LC network? We often understand it from the idea of *Q* (quality factor), which by definition is:

$$Q = \frac{Energy\_stored}{Energy\_dissipated} \quad \text{per unit time.} \qquad (1.46)$$

In resonating circuits, we can think *Q* as unloaded (just the internal RLC circuits) and loaded (together with the load to be driven by the resonating circuits).

Energy dissipation in RLC resonators:

- Resistive loss (including line resistance of L and C)
- Substrate loss (any conductor or semiconductors that have coupling to the magnetic fields of L such as the Eddy current)
- Radiation loss (such as in antenna, which means antenna can be represented by a resistor in the system circuits)
- Dielectric loss (dielectric of *C*, usually only serious when $f > 40$GHz)

The peak voltage at ω_o across *L* and *C* can be estimated as (RLC elements are in parallel)

$$V_{peak} = I_R \cdot R \qquad (1.47)$$

Therefore, the energy stored in the resonating *L* and *C* is:

$$E_{stored} = \frac{1}{2} C V_{peak}^2 = \frac{1}{2} C (I_R R)^2 \qquad (1.48)$$

The average power dissipated by the resistor is:

$$P_{dissipated} = \frac{1}{2} I_R^2 R \qquad (1.49)$$

Therefore, at ω_o

$$Q \equiv \omega_0 \frac{E_{stored}}{P_{dissipated}} = \frac{RC}{\sqrt{LC}} = \frac{R}{\sqrt{L/C}} = \frac{R}{Z_0} = \frac{R}{\omega_0 L} = \frac{R}{1/\omega_0 C} \qquad (1.50)$$

where $Z_0 = \sqrt{\frac{L}{C}} = \omega_0 L = \frac{\omega_0}{C}$ is called the characteristic impedance of the LC network.

We can now write the peak current magnitude in *L* and *C* at ω_o as:

$$|I_C| = |I_L| = \frac{V_{peak}}{Z_0} = \frac{|I_R| R}{\omega_0 L} = Q|I_R| = Q|I_{in}| \qquad (1.51)$$

We have achieved a large current amplification ONLY at ω_o !!!

1.12 The crystal radio receiver

Now you can understand how the crystal radio receiver, originally by Bose, worked back in 1897!

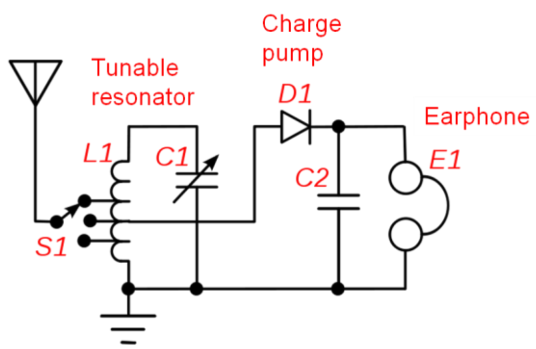


Fig. 1.22. Crystal radio receiver by Bose in 1897.

The antenna is the AC current source. After many cycles, the L_1/C_1 resonator achieves steady state and has a large current magnitude. If the “loaded” Q (including energy dissipated through diodes to actuate the earphone) is large, the positive cycles of the frequency-specific oscillations will be collected to C_2 and actuated the earphone. The diode with 0V turn-on voltage is critical, or else the AC voltage cannot be integrated into some useful DC potentials (the baseband of the crystal radio receiver). This is also a simple RFID frontend or class C power amplifier (often with much more complicated designs to boost the efficiency over 50%, and with various stages to pump the current/voltage higher than Q times).

In RF IC or PCB, the zero-threshold diode (or diode-connected transistor) is an important element. The threshold voltage needs to be at an optimal value. If the threshold voltage is too negative, the reverse leakage will reduce the efficiency. If the threshold voltage is too positive, only a small amount of energy with voltage above the threshold voltage can be scavenged.

Exercise:

For the series RLC network in Fig. 1.23, derive $\omega_o = \frac{1}{\sqrt{LC}}$, $Z_0 = \sqrt{\frac{L}{C}}$, $Q = \frac{\sqrt{L/C}}{R}$, and $\frac{|V_L|}{|V_{in}|} = Q$.

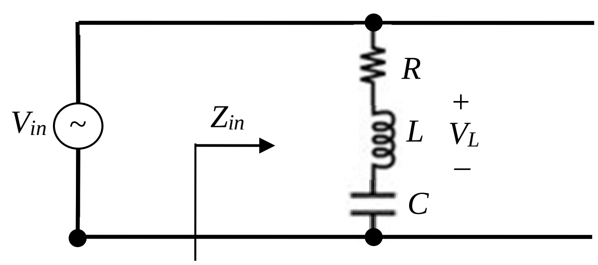


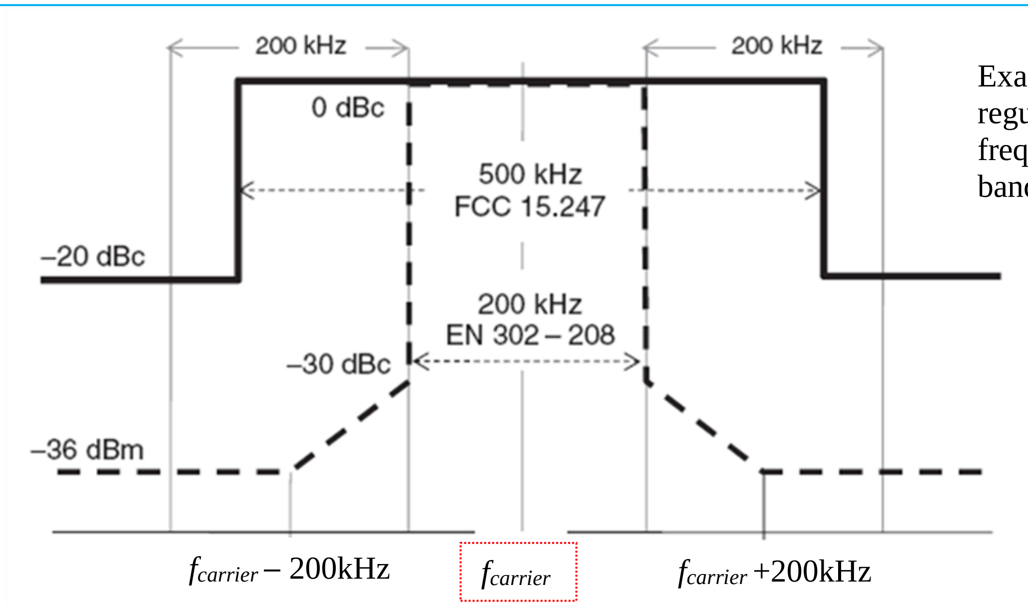
Fig. 1.23. An RF resonator with RLC in series.

1.13 Bandwidth and safety regulations for RF

Carriers as a single-tone do not carry identifiable information (it does carry the range information in its time-of-flight or phase shift). Any modulation in amplitude, phase, frequency or code will spread the spectrum usage a little bit. Let’s restrict ourselves first to this small frequency spreading around the center carrier frequency. Actually this will be most of the semester until we generalize to ultra-wide-band (UWB) radios, which we will NOT spend much time in this class. The bandwidth $BW = 2\Delta\omega$ is typically smaller than 1% of the carrier frequency, mostly for the sharing purposes. A typical example for the RFID regulation for *one channel* is shown below, where $f_{carrier}$ is around 866MHz in EU and 910MHz in US FCC (Federal Communication Commission). *BW* here is about 0.23% in EU and 0.55% in US in the respective carrier frequency.

To share the same “free” space for everyone, central regulation on frequency and power transmission is necessary for wireless communication. If the radio waves are confined in a cable or waveguide (such as cable TV and Internet, or in general, light in optical fibers), only the leakage out of the outer shield needs to be regulated. If the radio waves are confined in a chamber (such as your microwave oven with 2.45GHz operations), again only the leakage is important. Different regions may have different regulations in various frequency bands. Lack of global standards is an additional design burden for RF products, because some circuits need to be tuned to fit the different regulations in a specific country of operation.

The other concern is the human health and safety. In the bands between 30MHz to 1GHz, the OSHA (Occupation Safety and Health Administration) regulates the emission has to be lower than 1mW/cm^2 over a six-minute averaging period, mainly for the non-ionizing heat concerns. Higher frequency regulation can be found at OSHA web page:
https://www.osha.gov/SLTC/radiofrequencyradiation/electromagnetic_fieldmemo/electromagnetic.html



Example: RFID regulation on frequency and bandwidth

Fig. 1.24. An example of channel and bandwidth assignment by FCC.

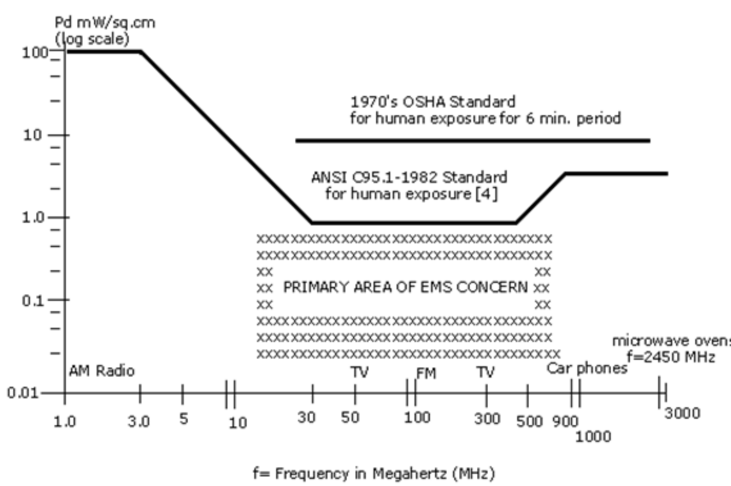


Fig. 1.25. OSHA RF safety regulation

1.13 Bandwidth and quality factor for resonators and filters

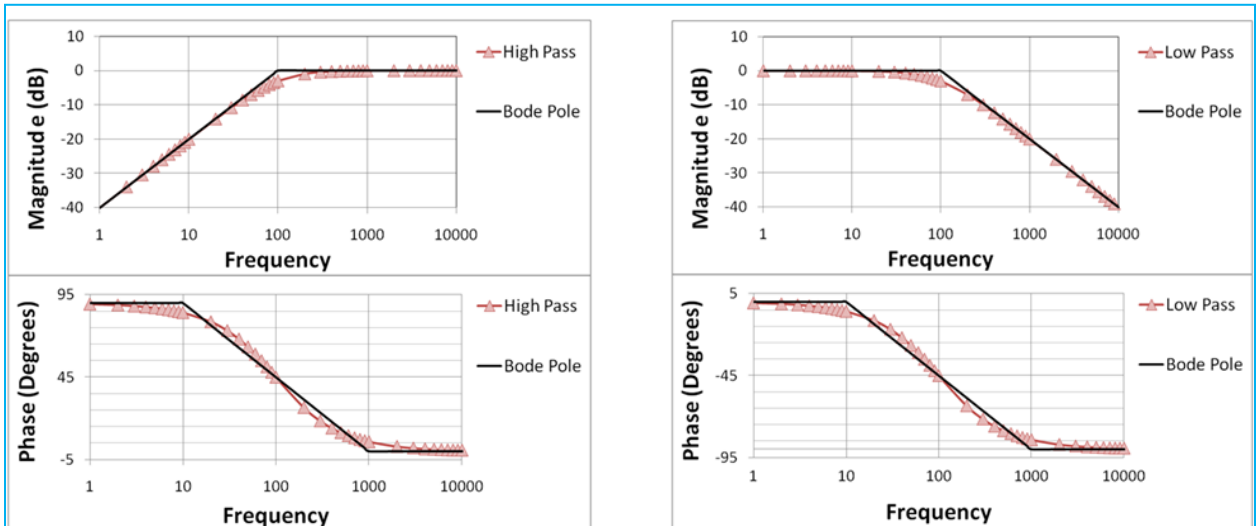
Let $\omega = \omega_0 + \Delta\omega$ where we assume $\Delta\omega << \omega_0$. The response from the above RLC network in parallel at ω_0 will then be

$$Y = \frac{1}{R} + \frac{j}{\omega L}\left(\omega^2 LC - 1\right) = \frac{1}{R} + \frac{j}{\omega L}\left(2\Delta\omega \cdot \omega_0 + \Delta\omega^2\right)LC \cong \frac{1}{R} + j\left(2C\Delta\omega\right) \qquad (1.52)$$

What does the imaginary term in *Y* mean? The behavior close to ω_0 is like the original resonator with a RC envelope with R in parallel with *2C*! We do know the 3dB bandwidth of a RC network is $\frac{1}{R \cdot C}$, so we can write (the bandwidth by $\Delta\omega << \omega_0$ needs to be 2 times):

$$\frac{BW}{\omega_0} = \frac{1}{RC\omega_0} = \frac{1}{Q}. \qquad (1.53)$$

For the resonating circuit, this makes sense. For large *Q*, *BW* will be correspondingly small, or in other words, the system is VERY frequency selective. A review of Bode plots for the low-pass, high-pass, and single-pole-zero circuits are shown below.



(a) $H_{high\_pass} = \dfrac{j\frac{f}{f_1}}{1 + j\frac{f}{f_1}}$ **(b)** $H_{low\_pass} = \dfrac{1}{1 + j\frac{f}{f_1}}$

Fig. 1.26. Bode plots of magnitude and phase for (a) high-pass and (b) low-ass filters

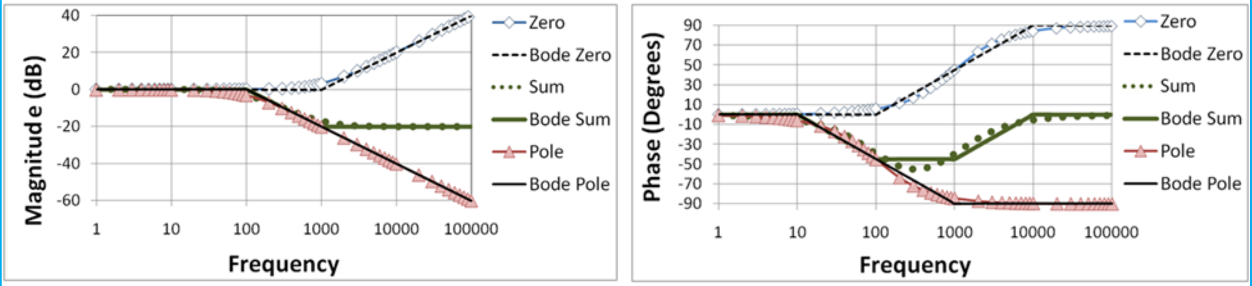


Fig. 1.27. The Bode plot of a one-pole and one-zero system where $f_{p1} < f_{z1}$. Both magnitude and phase for single-pole and single-zero cases are also shown.

High Q makes good frequency selection in a very precise resonant circuit. However, Q of an antenna can limit its BW, and for many channel hopping systems, the antenna needs reasonably large BW, and hence small Q. From the definition of Q, this also makes sense. The purpose of the antenna is to radiate EM energy to the far field (radiation loss can be treated as resistive loss in the transceiver system), instead of storing energy. Hence, if the antenna stores lots of energy around it (Q becomes large) instead of dissipating to free space, it is an antenna with large Q but very limited bandwidth. Notice that the antenna gain concerns with how the radiating energy is distributed in the far-field space, and has no direct relation with the antenna Q.

1.14 The conjugate impedance match

You have learned impedance matching in your introductory course. Impedance match has two purposes:

(1) Maximize power transfer given a source resistance (not maximizing signal amplitude);
(2) Minimize reflection in traveling waves (transmission lines, waveguides and antennas). This is critical for noise consideration, which we will treat details in the coming lectures.

Given a real source and load resistance in series (works in parallel as well by the Norton source), the power consumption at R_L can be expressed as:

$W_L = \dfrac{R_L V_S^2}{(R_S + R_L)^2}$. At a GIVEN R_S, to maximize W_L, we take the partial derivative of W_L with R_L while R_S is constant.

$$\left.\frac{\partial W_L}{\partial R_L}\right|_{R_S} = 0 \quad \Rightarrow \quad 0 = (R_S + R_L)^2 - 2R_L(R_S + R_L) \quad \Rightarrow \quad R_L = R_S \qquad (1.54)$$

As R_L and R_S have to be positive.

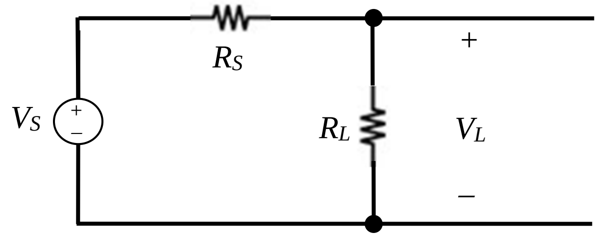


Fig. 1.28. Conjugate impedance match considerations.

Notice that the same derivation can be applied for complex impedance with a bit more complicated operation when $Z_S = R_S + jX_S$ and $Z_L = R_L + jX_L$. The partial derivative of W_L with Z_L while Z_S is constant will yield:

$$R_L = R_S;\ X_L = -X_S. \text{ Or } Z_L = Z_S^* \text{ for the \textbf{conjugate matching} condition.} \quad (1.55)$$

Also notice that when R_L is held constant, maximum of W_L gives $R_S = 0$!!! Not matching!!! For sure, this is natural as we hope R_L will take all the power delivered by V_S, when $V_L = V_S$. Also, this is just for maximizing W_L with no consideration of reflection for waves traveling from R_L (in transmission lines, this would be $R_L = Z_0$, the characteristic impedance) to R_S. The reflected wave can be seen as a noise addition to the voltage source, and is important in considering the transient of a digital link.

Exercise:

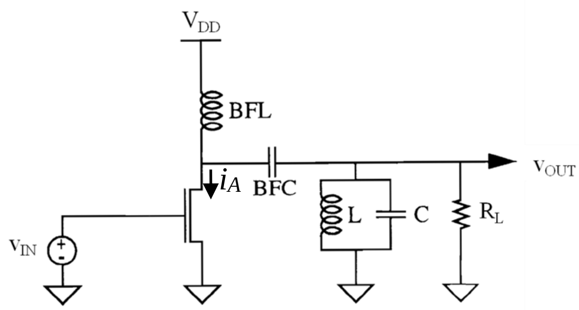
Someone tries to sell you an RF source (delivering say 1W in a tunable frequency range of 100M – 1GHz with nearly zero source impedance), claiming that the RF source by other companies with 50Ω source resistance can only deliver half of the power. What will be your response to the salesperson?

1.14.1 Impedance transfer from matching network

The LC bank is also popularly used as "matching network" to transform impedance. The other popular choice of matching network is the transmission line. By observation from our derivation up to now, the LC bank matching network will work ONLY close to the specific resonant frequency ω_0, and by definition, it is a narrow-band system. As the derivation is very similar to the previous resonant circuits, we will leave it to homework.

1.15 RF Power amplifiers

The resonator circuit is also a big part in the RF power amplifier. 1W of radiating power on a 50Ω antenna already has a peak-to-peak voltage of 7V, and the direct quasi-static power amplifier (such as a common-source or common-emitter amplifiier) is often not power efficient. A generic RF power amplifier is shown below with a RLC resonator output stage.



- BFL: big, fat inductor to keep current constant
- BFC: big, fat capacitor to block all DC leakage but pass all RF power
- R_L: load (most often, antenna or another PA)
- i_A: current injected by the active element

Fig. 1.29. Generic RF power amplifier (PA)

Class of an RF power amplifier is made on the operating point for the active elements. Class A has an active element that is always 100% on, Class B for only 50%, Class AB for ON between 50% and 100%, and Class C for less than 50%. Notice that the output full sinusoidal wave is maintained by the resonator circuit (which also determines the PA output frequency), and the active element is feeding the RF energy to the resonator by current injection of a much broader spectrum.

You can see that power amplifier only gives a significant RF output at the resonant frequency determined by *LC*. The RF power efficiency is measured by the radiating RF power vs. the input power of v_{IN} and V_{DD}. The transistor can burn significant power (the power in the transistor is wasted without being radiated), which determines the efficiency. Therefore, the design point in Class C PA with a "higher efficiency" is to turn on the transistor in smaller duty cycle, at the cost of the linearity. We will talk about the tradeoff between power efficiency and linearity later in the semester. Any power input to the PA without being radiated will become heat. For high-power transmission above 40dBm (10W), this can be significant. PA design indeed involves many temperature considerations, as i_A can be highly temperature dependent.

Except from parasitic elements not shown in the simple circuits and possible noise to the overall V_{DD} supply (power line noise), the RF PA is reasonably unilateral, i.e., signals travels from v_{IN} to v_{OUT}, not vice versa. However, in the transceiver circuits, even small leakage matters a lot. We will discuss about this later as well.

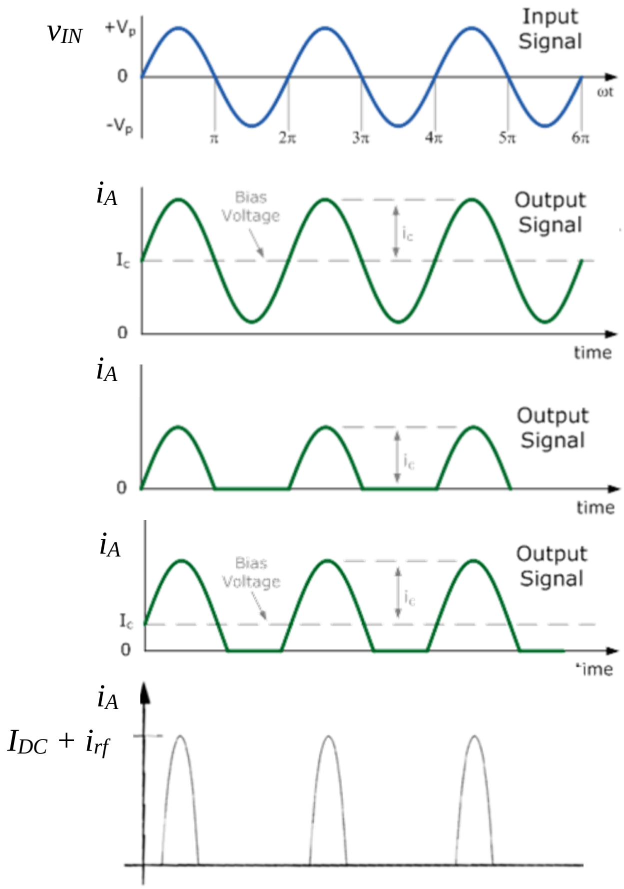


Fig. 1.30. Illustration of current injection into the LC resonator in Class A, B, AB and C RF power amplifiers

i_A in Class A RF amplifier

i_A in Class B RF amplifier

i_A in Class AB RF amplifier

i_A in Class C RF amplifier

A further note is probably worthwhile here. RF PA designs involve more than the simple resonators in-parallel (Class A, B, AB and C) or in-series (Class E) or combination (Class-F, involving harmonic cancellation), and deserve a full circuit treatment not covered in this course. However, there are two fundamental facts that we should know in the signal level:

1. RF PA relies on resonant circuits to maximize the current/signal swing in the antenna and hence free space, instead of just the basic amplifiers operating way below the cutoff frequency of the transistors. Therefore, it is important to consider the bandwidth of the power amplifier, and any potential contamination for signals in other frequency, whether this is in your own system, or this is another radio transceiver.

2. RF PA often sits at the interface between (a) the transceiver part that lumped circuit elements in SPICE (where KCL and KVL apply) give a good description and (b) the antenna part that ONLY Maxwell equations (integration of Hertzian dipoles) can explain the details. The simplified analogy of antenna to R_L in the circuit description remains only an analogy.

1.16 Signal-level representation of the RF power amplifier

For an amplifier in the radio signal chain, if the power level is below around 0dBm (for 1mW on a 50Ω load this corresponds to voltage level of 0.2V), we usually treat it as a broadband, quasi-static amplifier, where the amplifier is sufficiently described by its gain (current, voltage or transconductance), and I/O impedances. However, for RF PA, due to the use of the resonator in parallel or in series to boost the radiation efficiency in a particular band, additional parameters such as frequency, bandwidth, power efficiency and nonlinearity (described by IIP2 and IIP3, to be explained in system nonlinearity) are needed to represent sufficient information in the signal chain.

Any power input into the radio system that is not radiated by the antenna will eventually become heat to the system. At the RF PA, which is the major heat source of the radio system, the temperature coefficients and the means of heat dissipation are additional concerns. For example, if you need 40dBm (10W) transmission for a give range of 1km and the power efficiency of PA is 25% (class A), then at least 40W needs to be sent to the radio transmitter system with 30W dissipated as heat. We can use this rule of thumb of heat dissipation:

- ~1W: no special heat sink (your Bluetooth).
- ~10W: a static metallic sink (your cell phone).
- ~100W: an air fan or very good ventilation (your notebook computer, and base stations on lamp posts)
- >1kW: a liquid-cooling unit (radio/TV stations, radar, etc. Do not walk too close to the antenna!!!)