

# Inverse-Inverse Reinforcement Learning. How to Hide Strategy from an Adversarial Inverse Reinforcement Learner

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*Research funded by Lockheed Martin and the Army Research Office, presented at IEEE 61st Conference on Decision and Control (CDC), 2022.*

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**Main Idea.** Detecting utility maximization  $\equiv$  Checking linear feasibility  
*How to make checking linear feasibility difficult?*

## **Radar Context:**

Cognitive radar  $\rightarrow$  Choose optimal waveform for target tracking

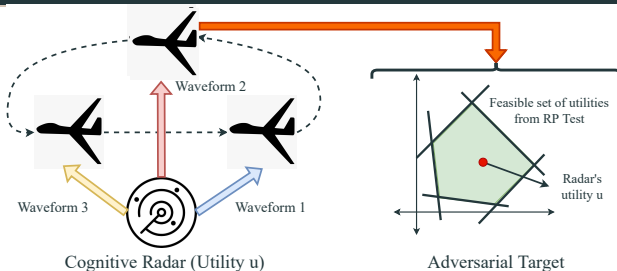
**Adversarial Target**  $\rightarrow$  Malicious maneuvers to 'estimate' radar's utility

*How to spoof adversarial attacks on radar's utility function?*

Ans. **Cognition Masking**

*Intelligently perturbed radar actions successfully hide radar's utility*

# Background. Cognitive Radar and Revealed Preference



**Cognitive Radar [1–3]: Optimal waveform adaptation.**

For target maneuvers (**probe**)  $\{\alpha_k\}_{k=1}^K$ , radar chooses waveforms (**response**)  $\{\beta_k\}_{k=1}^K$  that maximize utility  $u$ :

$$\beta_k = \operatorname{argmax}_{\beta \in \mathbb{R}_+^m} u(\beta), \quad \alpha'_k \beta \leq 1 \quad (1)$$

**Radar Bayesian tracker:** Linear Gaussian dynamics

- (i)  $\alpha_k$ : state noise covariance
- (ii)  $\beta_k$ : observation noise covariance
- (iii)  $\alpha'_k \beta_k \leq 1$  (1): Bound on radar SNR  $\equiv$  Bound on radar's asymptotic predicted Kalman **precision** [3]

*'Choose best waveform subject to resource constraints'*

**Utility Estimation via Revealed Preference (RP):**

**RP Test [4, 5]:** For dataset  $\mathbb{D} = \{u_k, \beta_k\}_{k=1}^K$ , linear feasibility test is **equivalent** to checking for utility maximization (1):

$$\text{RP}(u, \mathbb{D}) \leq 0, \quad u = \{u_k, \lambda_k\} \in \mathbb{R}_+^{2m}, \quad (2)$$

$$u_{\text{est}}(\beta) = \min_k \{u_k + \lambda_k \alpha'_k (\beta - \beta_k)\} \quad (3)$$

**What if  $\mathbb{D}$  is noisy?**

RP Test (2) generalizes to statistical hypothesis test to **detect** feasibility [6] (discussed in [slide 4](#)).

## Cognition Masking

How to mitigate adversarial RP test and ensure poor reconstruction of radar's utility function

# Result 1. Deterministic Inverse RP for Masking Cognition

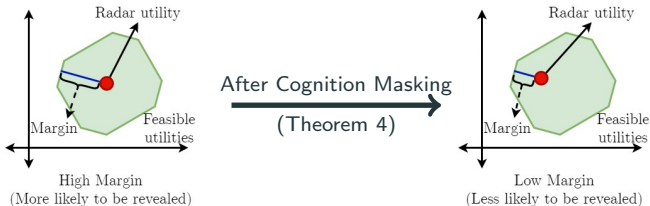
**Assumption:** “Radar and adversary have accurate probe-response measurements.”

Adversarial target  $\xrightarrow{\text{IRL}}$  RP Feasibility test (2) (Set-valued estimate of radar’s utility)

How to rank utility functions in the feasible set?

Rank via **Margin** of RP test - max. perturbation to fail RP test (based on [7])

$$\text{Margin}_{\mathbb{D}}(u) = \max_{\epsilon \geq 0} \epsilon, \text{ RP}(u, \mathbb{D}) + \epsilon \geq 0, u \in \text{Feasible set}$$



- Margin: Closeness to **edge of feasible set** (infeasibility of RP test)
- Center of feasible set: **max. margin**, edge of feasible set: **zero margin**
- $\uparrow$  Margin  $\iff$   $\uparrow$  Goodness-of-fit to RP test
- **Deterministic Cognition masking:** Deliberately perturb radar’s response to push radar’s utility **towards** edge of feasible set from RP test

## Deterministic Inverse IRL for Masking Cognition

Suppose radar faces adversarial constraints  $\{\alpha'_k \beta \leq 1\}_{k=1}^K$ . The radar's *deterministic* I-IRL algorithm to hide its utility  $u$  is:

**Step 1.** Choose margin  $\epsilon_{\text{thresh}} \in \mathbb{R}_+$

**Step 2.** Compute naive response  $\beta_k^*$  (1)

**Step 3.** Compute optimal perturbation  $\{\delta_k^*\}$  for I-IRL:

$$\{\delta_k^*\} = \underset{\{\delta_k\} \in \mathbb{R}^m}{\operatorname{argmin}} \underbrace{\sum_{k=1}^K \|\delta_k\|_2^2}_{\text{(Radar's degradation)}}, \underbrace{\operatorname{Margin}_{\{\alpha_k, \beta_k^* + \delta_k\}}(u) \leq \epsilon_{\text{thresh}}}_{\text{(Mitigating adversarial RP Test)}} \quad (4)$$

**Step 4.** Transmit engineered sub-optimal responses  $\{\beta_k^* + \delta_k^*\}$ .

## Summary

**Deterministic I-IRL:** Small margin  $\epsilon_{\text{thresh}}$

$\iff$  Closer to failing RP test (2)

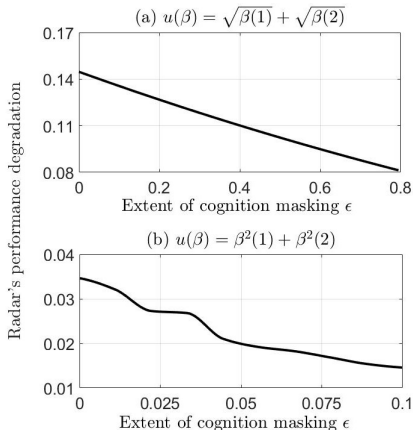
$\iff$  Larger deviation from radar's optimal strategy

- Margin Constraint in (4) is non-convex (bilinear).

**Current research:** *Formulate convex relaxations of bi-linear constraints in (4).*

# Numerical Results: Deterministic Inverse IRL

- Simulation-based datasets to illustrate I-IRL for 2 utility functions
- Parameters: Time horizon  $K = 50$ , Probe/Response dimension  $m = 2$



## Key Insights:

- **Small deviation** from *optimal strategy* masks utility by a large extent.
- Radar's performance degradation  $\uparrow$  with  $\epsilon$ .

## Result 2. Stochastic Inverse RP for Masking Cognition

**Assumption:** “Adversary has noisy measurements of the radar’s response.”

$$\text{(Adversary side): } \hat{\beta}_k = \beta_k + w_k, w_k \sim f_w \text{ (} f_w \text{ known to radar)} \quad (5)$$

Adversarial target  $\xrightarrow{\text{IRL}}$  **Feasibility Detector** (see also [3] for details)

$H_0$  : RP Test (2) has a feasible solution for  $\{\alpha_k, \beta_k\}$

$H_1$  : RP Test (2) has NO feasible solution for  $\{\alpha_k, \beta_k\}$

$$\text{IRL Feasibility Detector : } \phi^*(\hat{\mathbb{D}}) \leq_{H_0}^{H_1} F_L^{-1}(1 - \eta) \quad (\hat{\mathbb{D}} = \{\alpha_k, \hat{\beta}_k\}), \quad (6)$$

$$\phi^*(\hat{\mathbb{D}}) : \max_{\{\bar{u} > 0\}} \text{Margin}_{\bar{u}}(\hat{\mathbb{D}}), \text{ r.v. } L := \max_{j,k} \alpha_j'(w_j - w_k),$$

$\eta$  : Adversary chosen bound for  $\mathbb{P}(H_1|H_0)$

*“Radar is non-cognitive if margin is under a threshold”*

- Radar **can no more** manipulate margin of RP test.
- Can *at best* manipulate  $\mathbb{P}(H_1|\{\alpha_k, \beta_k\}, u)$  (**Cond. Type-I error prob.**)
- **Stochastic Cognition masking**: Deliberately perturb radar’s response to mitigate IRL detector (**increase** conditional Type-I error probability).

## Stochastic Inverse IRL for Masking Cognition

Adversary's sensor is noisy; everything else the same as deterministic case.  
 Radar's *stochastic* I-IRL algorithm is:

**Step 1.** Choose sensitivity parameter  $\lambda > 0$

**Step 2.** Compute naive response  $\beta_k^*$  (1)

**Step 3.** Compute optimal perturbation  $\{\delta_k^*\}$  for I-IRL:

$$\{\delta_k^*\} = \underset{\{\delta_k\} \in \mathbb{R}^m}{\operatorname{argmin}} \sum_{k=1}^K \left( \underbrace{u(\beta_k^*) - u(\beta_k^* + \delta_k)}_{\text{(Radar's deliberate performance loss)}} - \lambda \underbrace{\mathbb{P}(H_1 | \{\alpha_k, \beta_k^* + \delta_k\}, u)}_{\text{(Mitigating adversarial IRL detector)}} \right) \quad (7)$$

**Step 4.** Transmit engineered sub-optimal responses  $\{\beta_k^* + \delta_k^*\}$

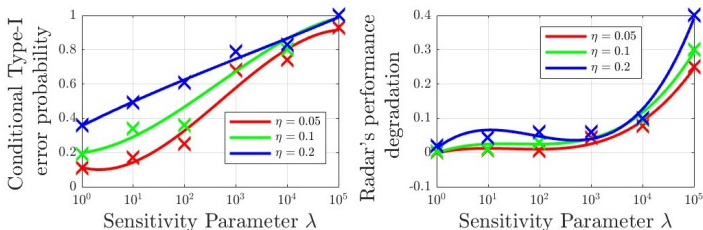
(7): Ensuring low margin of RP Test with high probability

## Summary

- **Stochastic I-IRL:** Trade-off between  $\uparrow$  QoS and  $\uparrow$  *adversarial obfuscation*.
- Radar can only estimate  $\mathbb{P}(H_1 | H_0, u)$  (7) via Monte-Carlo methods.
- Stochastic approximation based algorithms like **SPSA [8]** can be used for implementing optimization problem (7).
- SPSA  $\rightarrow$  Fewer (only 2) computations/update wrt finite diff. methods.

# Numerical Results: Stochastic Inverse IRL

- Simulations for a single utility function  $u(\beta) = \sqrt{\beta_1} + \sqrt{\beta_2}$
- Parameters: Time horizon  $K = 50$ , Probe/Response dimension  $m = 2$



## Key Insights:

- **Small performance loss** sufficiently confuses IRL detector (**large cond. Type-I error**).
- **Both** adversarial confusion and radar's performance degradation  $\uparrow$  with  $\lambda$ .
- Interestingly, performance degradation  $\downarrow$  with  $\eta$  (error bound).

**Remark:** Inverse IRL results on slides 3,6 can be extended to the case where radar hides its system constraints and adversary dictates the radar's utility function, for e.g., beam allocation (Th. 3 in paper).



## Result 3. Finite Sample Effects for Inverse IRL

Stochastic I-IRL (slide 6) adapts deterministic I-IRL to strategy 'detector'.

**Key Idea.** Sufficient statistics for existence of strategy in terms of observation noise.

*What if radar has noisy measurements of the adversary's probes?*

**Prob. bounds for Deterministic I-IRL (slide 3) to mask strategy effectively?**

Recall: Deterministic I-IRL maintains feasibility margin of IRL test **less than**  $\epsilon_{\text{thresh}}$  (4).

Want to bound:  $\mathbb{P}(\text{Margin}_{\{\alpha_k + w_k, \tilde{\beta}_k^*\}}(u) \not\leq \epsilon_{\text{thresh}})$ , where  $w_k \rightarrow$  Radar sensor's measurement noise,  $\tilde{\beta}_k^* \rightarrow$  I-IRL response (4). Assume i.i.d  $w_k \sim \mathcal{N}(\mathbf{0}, \Sigma)$ .

### Finite Sample Complexity for Deterministic I-IRL

Consider the radar choosing I-IRL responses according to (4) and observes adversary's probes in noise. Then, under mild conditions, the probability that deterministic I-IRL fails to mask the radar's strategy is given by:

$$\mathbb{P}(\text{Margin}_{\{\alpha_k + w_k, \tilde{\beta}_k^*\}}(u) > \epsilon_{\text{thresh}}) \leq 1 - \frac{T e^{-\psi^2(\hat{\mathbb{D}})/2}}{\psi(\hat{\mathbb{D}})\sqrt{2\pi}}, \quad \hat{\mathbb{D}} = \{\alpha_k + w_k, \beta_k\}_{k=1}^T,$$

$\psi(\cdot)$  (8) is proportional to Lipschitz constant of radar's constraint, range of allowable probes, and inversely proportional to Lipschitz constant of radar's utility function.

**Remark.** Above error bound is loose, currently investigating tighter convergence rates.

# Conclusion and Extensions

## Summary:

- Radar **counter**-countermeasure to mitigate an adversarial countermeasure
- Cognition Masking: *Deliberately perturb optimal radar waveforms to sufficiently reduce margin of RP test and 'hide' radar's utility.*
- Sub-optimality in response trades-off between **Privacy** and **Performance**
- Methodology inspired from adversarial obfuscation [9] in deep learning and differential privacy [10]

## Extensions (Current research):

1. *Online IRL*. Current strategy hiding idea is offline (since IRL via Afriat's Theorem is intrinsically offline). Bandit approach for approximating IRL detector?
2. *Meta-confusion*. Vary the low margin constraint over time for 'robust' adversarial mitigation.
3. *Semi-parametric*. Jointly optimize over response perturbations and variance of additive Laplacian noise for robust I-IRL.
4. **Counter**-(counter-)<sup>n</sup>measure: What if adversary knows radar's spoofing strategy? *Game theoretic approach?*

Thank You!

# Miscellaneous

- **How justified is the constrained utility maximization abstraction for radar operation?**

**Quite prevalent in literature:**

- (i) Multi-UAV network [11]: **Utility** → Fairness and downlink data rate, **Constraint** → Transmission power, Cramer-Rao bound on localization accuracy
- (ii) Q-RAM (Resource Allocation) [12]: **Utility** → QoS for tracking and search, **Constraint** → Bandwidth, Short-term and Long-term constraints
- (iii) Radar Tracking with ECM [13]: **Utility** → Neg. of weighted mean of radar energy and dwell time, **Constraint** → 4% Cap on lost tracks due to ECM

- **Is conditional Type-I probability the only I-IRL metric for adversarial obfuscation in stochastic I-IRL?**

**No fixed formula, does need more work.** Some intuitive alternatives: (a) Use deterministic I-IRL as is. Formulate concentration inequalities for margin of the noisy dataset.

(b) Manipulate the average margin instead of margin. BUT, might be underplaying robustness of IRL detector.

(c) [**Speculative**] Use a neural network to learn IRL method on the fly and disrupt ECM.

*Remark: I-IRL hinges delicately on IRL methodology.*

*Other heuristic ideas to hide utility?*

- **What's next after IRL, and inverse IRL? I2-IRL?**

Game-theoretic formulation.

Key challenge: Formulate a utility function in terms of both adversary probes and radar response.

*Anticipated outcome:* Inverse game theory - Detecting play from the Nash equilibrium of a game between adversary and radar.

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