

Task 2 – Four-Point Bending of Bionax Pipe and Pipe Joints

Submitted to:

Jeff Phillips
Western Regional Engineer
IPEX Management, Inc.
20460 Duncan Way
Langley, BC, Canada V3A 7A3
Ph: 604-534-8631
Fax: 604-534-7616
e-mail: Jeff.Phillips@ipexna.com

Submitted by:

School of Civil and Environmental Engineering
Cornell University
Hollister Hall
Ithaca, NY 14853
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1. Introduction

This report is the final in a series of three reports to IPEX Management, Inc. and describes the results of Task 2 – Four-Point Bending of Bionax Pipe and Pipe Joints. The report presents the results of rotation tests on Bionax pipe segments with a restrained bell-and-spigot joint and another specimen of straight Bionax pipe without a joint. All testing was performed at the Cornell University Large Scale Lifelines Testing Facility. IPEX provided all pipe materials and connection fittings. The primary pipe tested was Bionax 6 in. (150 mm) CIOD. The joint restraints were Model UFR1559-C-6-I restraints for C909 PVCO, manufactured by the Ford Meter Box Co., Inc.

2. Straight Pipe

Test RT2 was performed on a straight section of Bionax 6 in. (150 mm) CIOD pipe. Table 2.1 lists the primary instrumentation types, names, and locations. Figure 2.1 is a schematic of the RT2 test layout. The pipe was pressurized to approximately 75 psi (517 KPa.) The test specimen was supported on rollers at the ends and loaded at the one-third points using a spreader beam. The distance between the south end support and the first load point was 28 in. (711.2 mm); the distance between the two central load points was also 28 in. (711.2 mm); and the distance between the second central load point and the north support also 28 in. (711.2 mm). Thus, the moment arms for uniform bending in the central portion of the pipe section were both 28 in. (711.2 mm)

The primary instrumentation used to determine rotations in RT2 test consisted of:

- a) Three string potentiometers (string pots) located at the center load points (pots D and F) and at the pipe centerline (pot E.) These string pots were connected between the pipe invert and the test floor. Pots D and F are located roughly ± 13 in. (± 330 mm) from the pipe center
- b) Actuator displacement, which was equivalent to the displacement at the two central load points. These locations are ± 14 in. (± 355.6 mm) from the center of the specimen.
- c) Total applied load, P. This load is equally applied at the two central load points, making the load at each point $P/2$. Consequently, the applied uniform moment between the load points is $P/2 \times (L/3)$ [where L is equal to 84 in. (213.4 cm), the total distance between the north and south supports.]

Table 2.1. Instrumentation for IPEX Bionax Rotation Test RT2

| Location | Instruments | Local Instrument Name |
|-----------------------|--|---|
| -13.1 in. (-332.7mm) | String Potentiometer on Pipe Invert | Pot D |
| 0 | String Potentiometer on Pipe Invert | Pot E |
| +13.3 in. (+337.8 mm) | String Potentiometer on Pipe Invert | Pot F |
| -28 in. (-711.2 mm) | Axial Strain Gages at Crown and Invert | (Gage Plane A) -28 Crown -28 Invert |
| 0 | Axial Strain Gages at Crown and Invert | (Gage Plane B) 0 Crown 0 Invert |
| +28 in. (+711.2 mm) | Axial Strain Gages at Crown and Invert | (Gage Plane C) +28 Crown +28 Invert |
| ±14 in. (±355.6 mm) | Actuator Displacement | Act. Disp. |
| 0 | Load Cell | Load |

d) Three strain gage planes: Planes A and C located at ±28 in. (±711.2 mm) from the center and Plane B at the center of the pipe.

Figure 2.2 shows the measured actuator force vs. the actuator displacement. Due to the unusual actuator force measurements in the initial portion of the curve, both on loading and unloading, the strains at gage planes A, B, and C were used to determine the actual loads required to generate the measured strains. This approach is described in Appendix A. Figure 2.3 compares the measured and calculated actuator force vs. actuator displacement. The calculated forces are substantially greater than those measured, and also linear with displacement, as expected for this test. For reliable test results the loads determined from strain gage measurements, as shown in Figure 2.3, are used in this report to evaluate the RT2 test data.

Figure 2.4 shows the string pot displacements at the pipe invert at approximately the ±14 in. (±355.6 mm) load points vs. the actuator displacements on the pipe crown at these locations. The string pots and actuator displacements are in very good agreement.

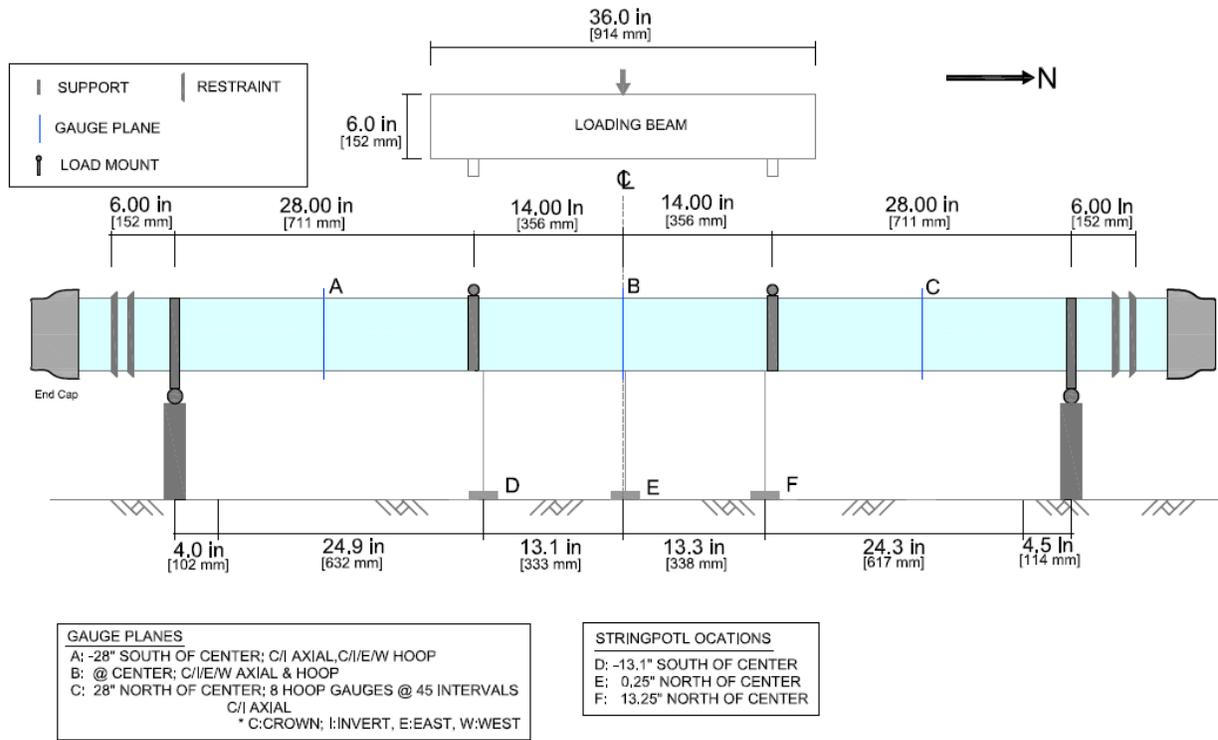


Figure 2.1. Instrumentation Schematic of Rotation Test RT2

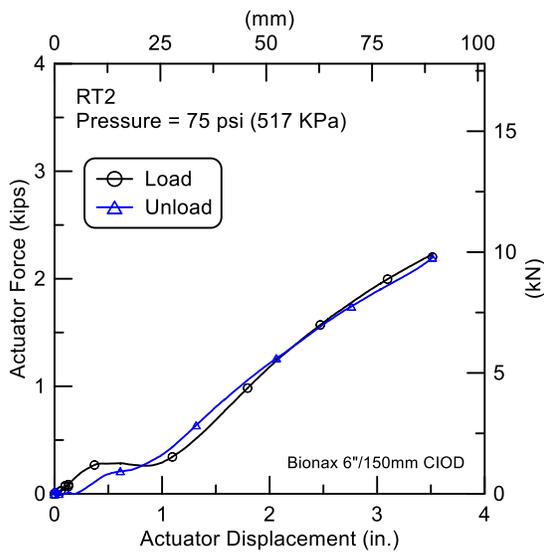


Figure 2.2. RT2 Measured Actuator Force vs. Actuator Displacement

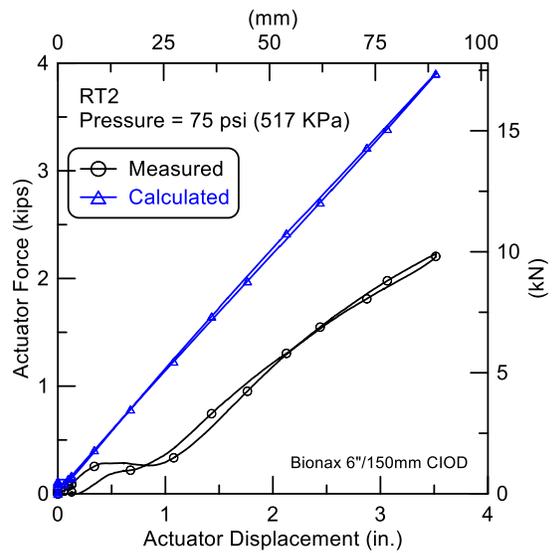


Figure 2.3. RT2 Measured and Calculated Actuator Forces

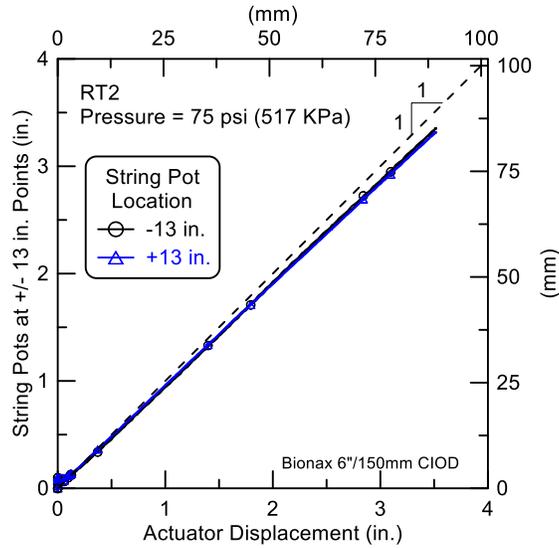
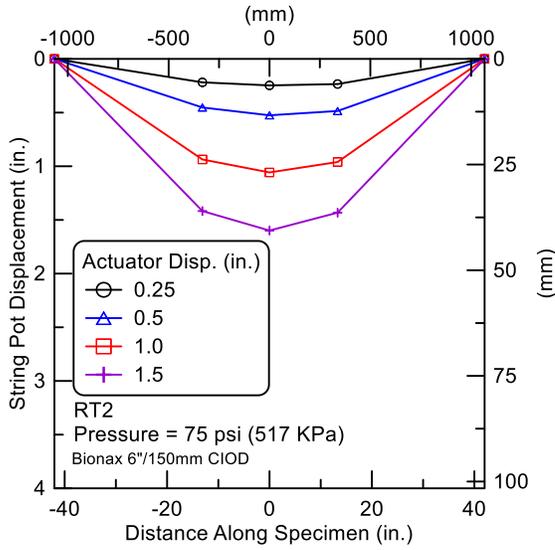


Figure 2.4. RT2 String Pots vs. Actuator Displacement

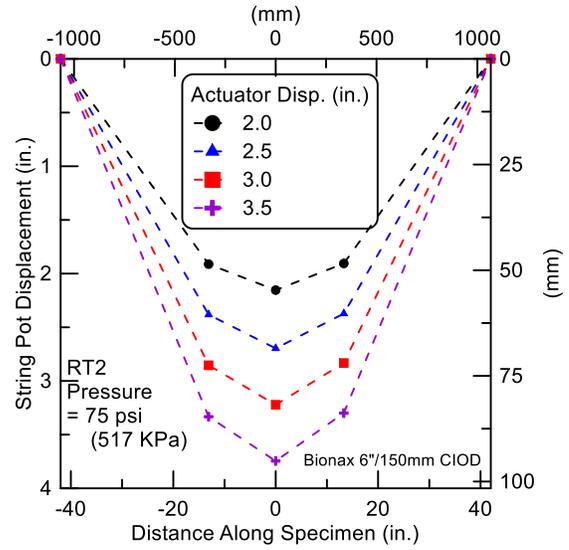
At an actuator displacement of 3.5 in. (88.9 mm) the string pots at the pipe invert show movements of 0.15 in. (3.81 mm) less than at the crown. This difference is a result of local deformation of the pipe at the load points, causing the crown to deform slightly more than the invert.

The vertical displacements measured at the three string pot locations along the pipe invert are shown in Figure 2.5 with respect to varying levels of actuator displacement. These measurements show a smooth, continuous increase in displacements at all locations. In addition, vertical displacements at the ± 13 in. (± 330 mm) locations are nearly identical for each actuator displacement

The crown and invert axial strains at the pipe center (0) and ± 28 in. (± 711.2 mm) vs. actuator displacement are shown in Figure 2.6. Crown and invert strains are compressive and tensile, respectively. The strains at the ± 28 in. (± 711.2 mm) are virtually identical, and roughly one-half those at the pipe crown and invert. The applied moment at the ± 28 in. (± 711.2 mm) points is one-half that at the pipe center, so the strains should be one-half those at the center, assuming the neutral axis does not change during loading. Figure 2.7 shows the strains at these three gage planes as dependent on applied moment. Again, the strains at all locations are proportional to moment.



a) 0.25 to 1.5 in.



b) 2.0 to 3.5 in.

Figure 2.5. RT2 String Pot Displacements and Actuator Displacement vs. Distance along Pipe

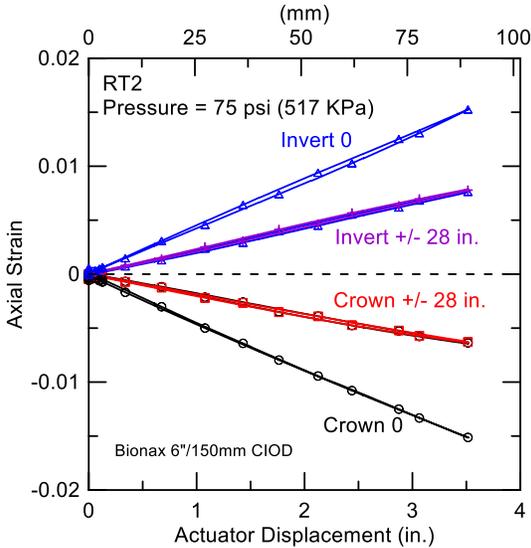


Figure 2.6. RT2 Strains vs. Actuator Displacement

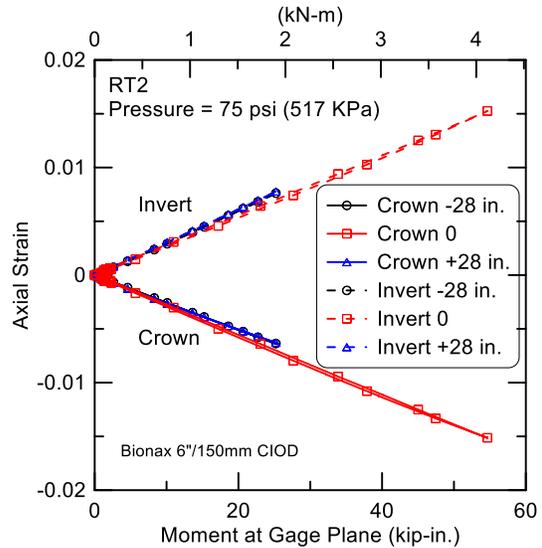


Figure 2.7. RT2 Axial Strains at Gage Planes A, B, and C

The rotations in the central portion of the straight pipe can be calculated using the three string pots; location D [-13.1 in. (-332.7 mm)], location E (center) and location F [+13.3 in. (+337.8 mm).] The relative deflection between pot E and the center is designated δ_{South} and the relative

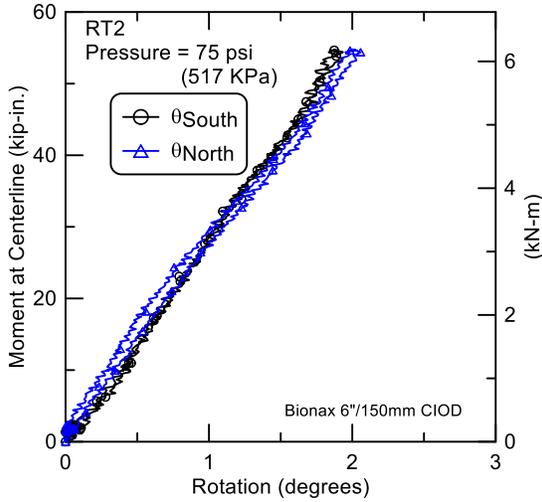


Figure 2.8. RT2 South and North Side Relative Rotations

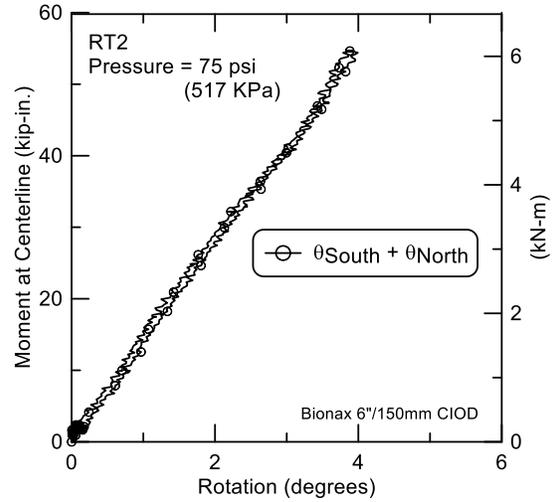


Figure 2.9. RT2 Rotation vs. Moment

deflection between pot F and the center is designated δ_{North} . The angles, θ_{South} and θ_{North} , between the outer pots (D and F) and the center (E) are:

$$\begin{aligned} \theta_{\text{South}} &= \tan^{-1}(\delta_{\text{South}} / 13.1 \text{ in.}) \text{ and} \\ \theta_{\text{North}} &= \tan^{-1}(\delta_{\text{North}} / 13.3 \text{ in.}) \end{aligned} \quad (1)$$

Figure 2.8 shows the rotation angles and moments in the central portion of RT2. Each side of the pipe rotated correspondingly, as expected. The equivalent pipe rotation in the central portion of the straight pipe, between the loading points, is the sum of the two rotation angles. These combined rotations are shown in Figure 2.9.

3. Restrained Joint

Test RT3 was performed on a section of Bionax 6 in. (150 mm) CIOD pipe with a restrained joint in the center. The pipe was pressurized to approximately 75 psi (517 KPa.) The test specimen was supported on rollers at the ends and loaded at the one-third points using a spreader beam. The geometric set up for this test was the same as for Test RT2. Figure 3.1 is a schematic of the RT3 test layout. Table 3.1 lists the primary instrumentation types, names, and locations used to determine rotations in Test RT3. The instrumentation consisted of:

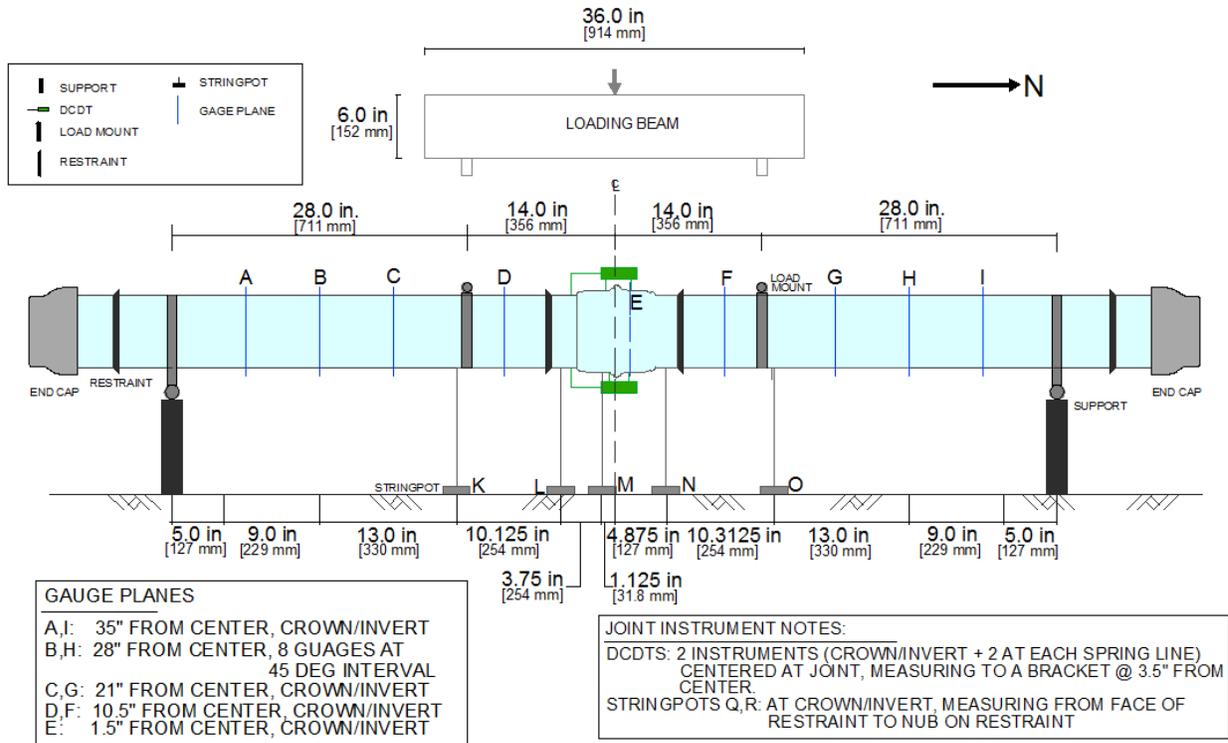


Figure 3.1. Instrumentation Schematic of Rotation Test RT3

Table 3.1. Instrumentation for IPEX Bionax Rotation Test RT3

| Location | Instruments | Local Instrument Name |
|------------------------|--|--|
| -15 in. (-381.0 mm) | String Potentiometers on Pipe Invert | Pot K |
| -5 in. (-127.0 mm) | String Potentiometers on Pipe Invert | Pot L |
| -1.125 in. (-28.6 mm) | String Potentiometers on Pipe Invert | Pot M |
| +4.875 in. (+123.8 mm) | String Potentiometers on Pipe Invert | Pot N |
| +15.2 in. (+386.1 mm) | String Potentiometers on Pipe Invert | Pot O |
| ±35 in. (±889.0mm) | Axial Strain Gages at Crown and Invert | (Gage Planes A and I) ±35 Crown ±35 Invert |
| ±28 in. (±711.2 mm) | Axial Strain Gages at Crown and Invert | (Gage Planes B and H) ±28 Crown ±28 Invert |
| ±21 in. (±533.4 mm) | Axial Strain Gages at Crown and Invert | (Gage Planes C and G) ±21 Crown ±21 Invert |
| ±10.5 in. (±266.7 mm) | Axial Strain Gages at Crown and Invert | (Gage Planes D and F) ±10.5 Crown ±10.5 Invert |
| ±14 in. (±355.6 mm) | Actuator Displacement | Act. Disp. |
| 0 | Load Cell | Load |
| 0 | DCDTs at Crown and Invert | DCDT C & I |

- a) Five string potentiometers (string pots) located along the pipe invert. These string pots were connected between the pipe invert and the test floor. See Table 3.1 for these string pot locations.
- b) Actuator displacement, which was equivalent to the displacement of the pipe crown at the two central load points. These locations are ±14 in. (±355.6 mm) from the center of the specimen.
- c) Total applied load, P. This load is equally applied at the two central load points, making the load at each point P/2. Consequently, the applied uniform moment between the load points is $P/2 \times (L/3)$ [where L is equal to 84 in. (213.4 cm), the total distance between the north and south supports.]
- d) Eight gage planes (A thru D and F thru I.). See Table 3.1 for gage plane locations
- e) DCDTs located above the crown and below the invert at the joint.

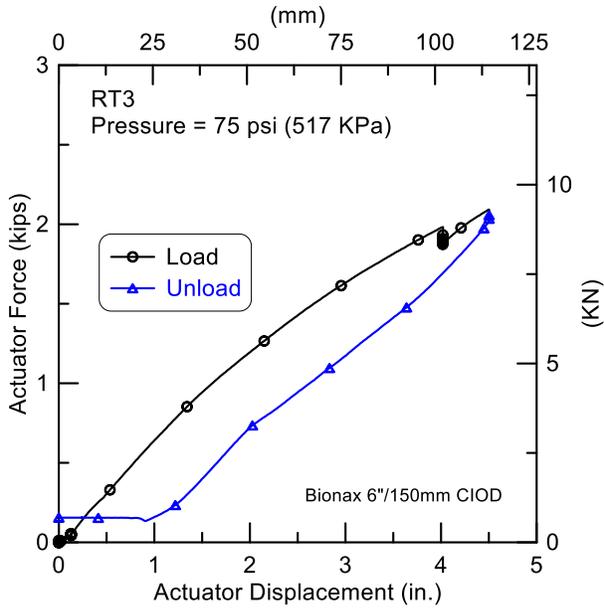


Figure 3.2. RT3 Actuator Force vs. Actuator Displacement

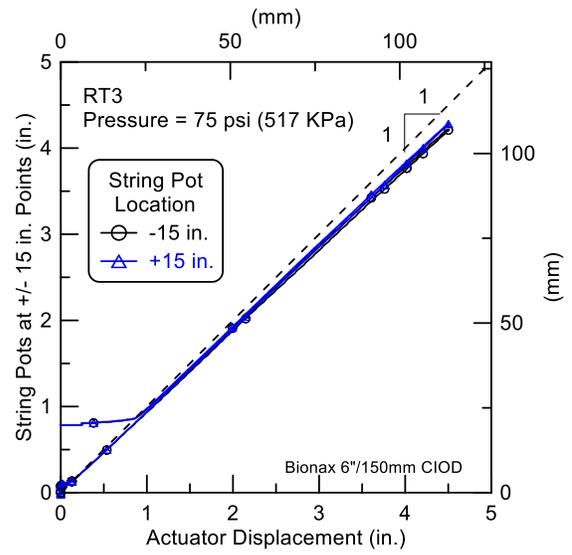
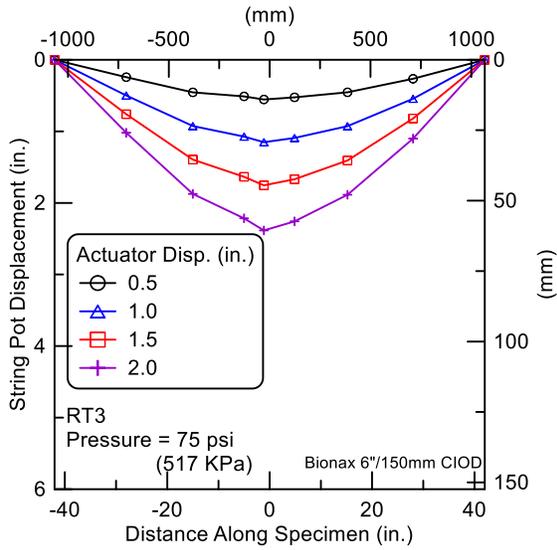
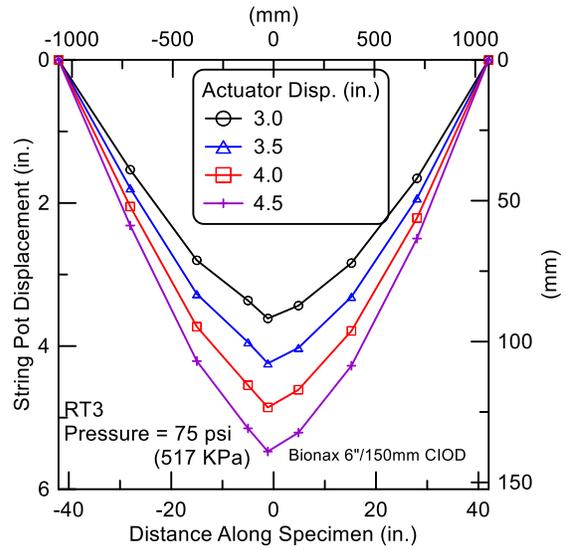


Figure 3.3. RT3 String Pots vs. Actuator Displacement



a) 0.5 to 2.0 in.



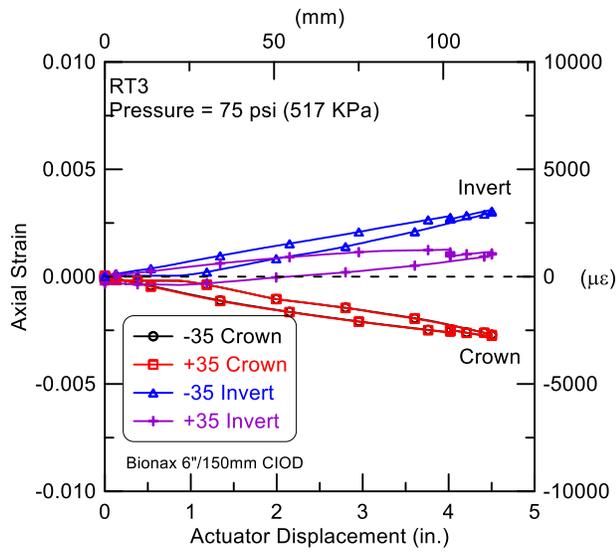
b) 3.0 to 4.5 in.

Figure 3.4. RT3 String Pot Displacements and Actuator Displacement vs. Distance along Pipe

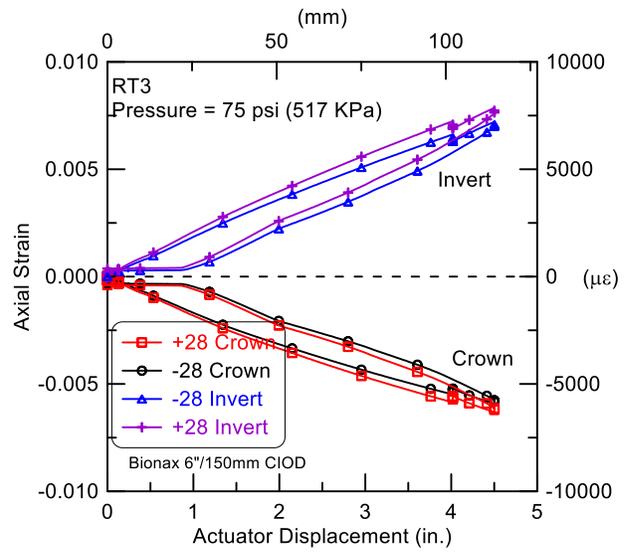
Figure 3.2 shows the total applied actuator force vs. actuator displacement. Figure 3.3 shows the string pot displacements on the pipe invert at distances ± 15 in. (± 381.0 mm) from center vs. the actuator displacements, which were applied at the ± 14 in. (± 355.6 mm) load points at the pipe crown. Again, the string pot displacements are slightly less than the actuator displacements due to additional local deformations at the load points. The residual displacement of about 0.75 in. (19.1 mm) shown in Figure 3.3 occurs during the unloading phase of the test.

The movements measured at the string pot locations along the pipe invert are shown in Figure 3.4 as a function of distance along the specimen. These measurements show a smooth, continuous increase in displacements at all locations.

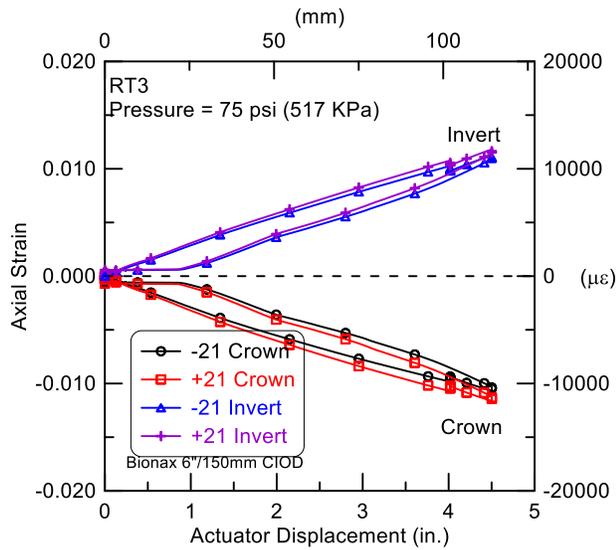
The strains at the pipe crown and invert increase with actuator displacement, as shown in Figures 3.5 a-d. With the exception of the invert locations at ± 35 in. (± 889.0 mm) the strains at the crown and invert are symmetric. This pattern also is shown in Figure 3.6, which plots the strains along the test specimen vs. actuator displacement. The instrumentation shows nearly a linear increase in strain as the load points are approached. Because the moment increases linearly with distance from the end supports, a linear increase in strain is fully consistent with the bending moment distribution.



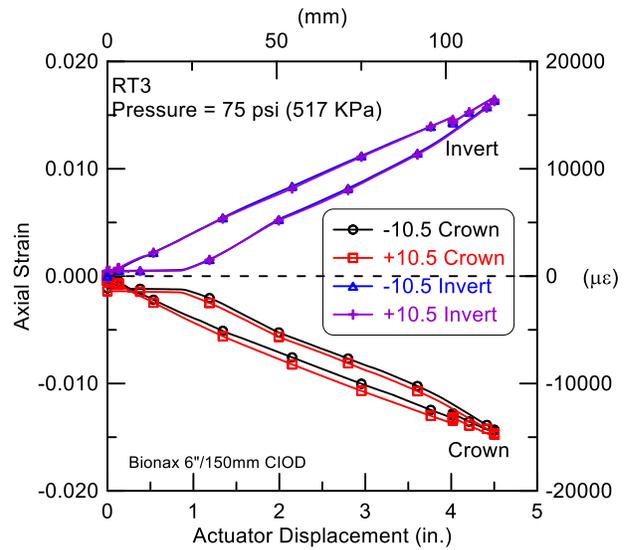
a) Gage Planes A and I ± 35 in. (± 889 mm)]



b) Gage Planes B and H ± 28 in. (± 711 mm)]



c) Gage Planes C and G ± 21 in. (± 533 mm)]



d) Gage Planes D and F ± 10.5 in. (± 237 mm)]

Figure 3.5. RT3 Strains at Pipe Crown and Invert vs. Actuator Displacement

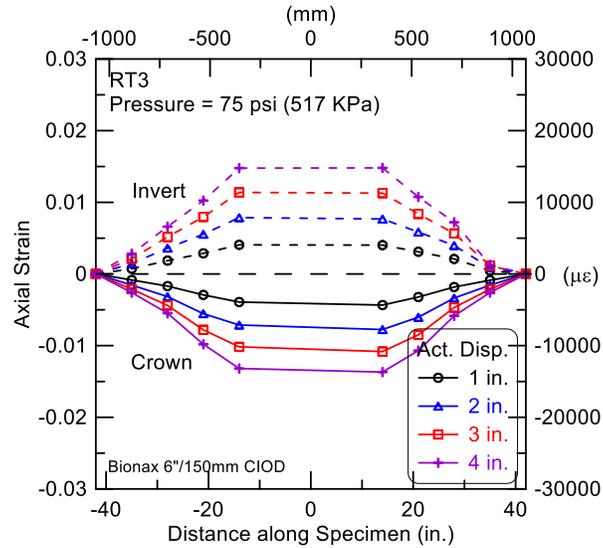


Figure 3.6. RT3 Axial Strains at Crown and Invert vs. Distance along Pipe

The rotations in the area of the restraint for RT3 were calculated three ways. The first was to determine joint rotation directly using the DCDT (LVDT) measurements at the pipe crown and invert. The second was to calculate the rotations of the pipe based on the string pot measurements of both Tests RT2 and RT3 at the pipe invert near the load points. The third was to use the rotations determined from the actuator displacements at the load points. The three methods all provide similar results.

The calculation procedure for rotation was similar to that described previously. The crown and invert DCDT separation distances in combination with displacement measurements provide a determination of joint rotation. The relative rotation of the south and north central sections were obtained using the string pot and actuator displacements, instrument locations, and pipe center displacement. These rotations were then added together to obtain the pipe rotation in the central zone.

The joint rotation during Test RT3 is determined using either the string pot measurements of invert rotation or rotation derived from actuator displacement at the pipe crown from both Tests RT2 and RT3. Here the pipe rotations from RT2 (calculated using either the

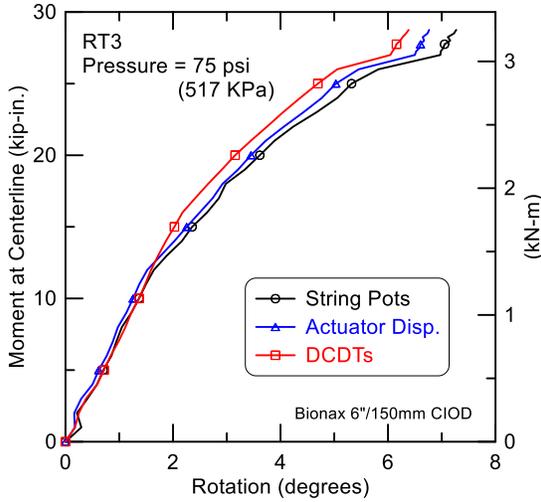


Figure 3.7. RT3 Joint Rotations

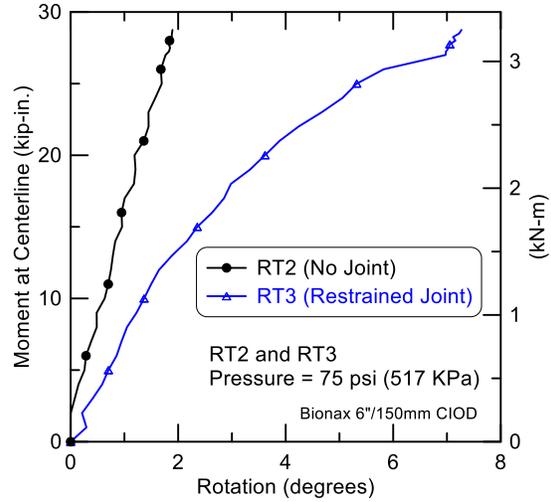


Figure 4.1. RT2 and RT3 Rotation Relations

relative displacements of the string pots or the actuator movements at the load points) were subtracted from those of RT3 (using either the relative displacements of the load point string pots or the actuator movements at the load points) to determine the “joint only” rotation. The rotations of the joint for RT3 based on each of the three measurement methods are shown in Figure 3.7.

4.0 Summary

Four-point bending tests on nominal 6-in. (150-mm)-diameter Bionax pipe were performed. Both pipe sections were pressurized to 75 psi (517 kPa) internal water pressure. Test RT2 was performed on a straight, unjointed section of pipe. Test RT3 was performed on a jointed section using a standard restraint. Forces were applied using a hydraulic actuator and pipe response was measured using various types of instrumentation, including load cells, DCDTs, string potentiometers, and strain gages. Appendix B contains several photographs of the experimental apparatus and specimens used for the tests.

Figure 4.1 compares the moment-rotation relationships for both an unjointed and a restrained, jointed pipe. The test results show that the restraint used with the Bionax pipe can accommodate substantial deformation through rotation at the joints. The restrained pipe is substantially more flexible in direct bending than the unjointed pipe. A comparison at similar bending moments shows that the restrained joint exhibits substantial nonlinear behavior with rapidly increasing rotation in response to increased moment. The relatively flexible response of the restrained joint will allow Bionax pipe with similar restrained joints to accommodate earthquake-induced ground

Figure 4.1 compares the moment-rotation relationships for unjointed and restrained, jointed pipe. The restrained pipe is substantially more flexible in direct bending than the unjointed pipe. A comparison at similar bending moments shows that the restrained joint exhibits substantial nonlinear behavior, with rapidly increasing rotation in response to increased moment. The relatively flexible response of the restrained joint will allow Bionax pipe with similar restrained joints to accommodate earthquake-induced ground deformation with relatively large rotation at low to moderate levels of moment. This type of response helps the jointed pipe to adjust to ground deformation perpendicular to the longitudinal axis of the pipeline at relatively low levels of bending stress in the pipe.

References

AISC Steel Construction Manual, American Institute of Steel Construction, Inc. Thirteenth Ed., 2005.

Appendix A – RT2 Loads

This appendix presents the method used to adjust the measure loads for Test RT2. The load cell measurements for this test showed unusual behavior in the early loading and late unloading portions of the test not seen in any prior tests. The strain gage data at several locations gave very repeatable and consistent measurements, and were used to calculate the required actuator loads necessary to generate the measured strains. This essentially requires using several strain gage planes and locations as load cell.

Axial strains were measured at the crown and invert of gage planes A, B, and C. These planes were located -28 in. (-711.2 mm) south of the pipe center (A), pipe center (B) and +28 in. (+711.2 mm) north of the pipe center (C). Gage planes A and C are midway between the pipe end supports and the central load points. The span between the central load points is the same as the distance from the support points to the load points, so the four-point bending test is completely symmetric.

Figure A.1 shows the stains measured at the crown and invert for RT2. The crown and invert strain at the pipe centerline (Plane A) are nearly identical in magnitude. At both Planes A and C the crown strains are smaller than the invert strains, but the magnitudes of all the strains at these gage planes are the same.

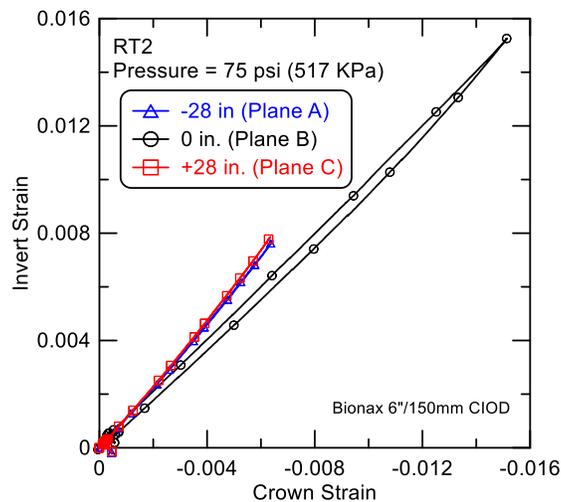


Figure A.1. RT2 Strains at Crown and Invert

The procedure for evaluating the applied load for RT2 from the crown and invert strains at three gage planes (assuming that the neutral axis of the pipe does not change during bending) is as follows. Due to the close correlation and symmetry among gage measurements, this approach averages strains at equal distances for pipe center.

1. Calculate the average strain at the pipe centerline, ε_0 , and average strain at ± 28 in. (± 711.2 mm), ε_{28} , as:

$$\varepsilon_0 = \frac{|\varepsilon_{0\text{crown}}| + |\varepsilon_{0\text{invert}}|}{2} \text{ and} \quad (\text{A.1a})$$

$$\varepsilon_{28} = \frac{|\varepsilon_{-28\text{crown}}| + |\varepsilon_{-28\text{invert}}| + |\varepsilon_{28\text{crown}}| + |\varepsilon_{28\text{invert}}|}{4} \quad (\text{A.1b})$$

2. Determine the average axial stress in the pipe as

$$\sigma_0 = \varepsilon_0 E \text{ and } \sigma_{28} = \varepsilon_{28} E \quad (\text{A.2a})$$

Use $E = 437$ ksi, (3.01 GPa) as reported previously. (Note that this does not correct for modulus reduction at strains greater the 0.007.)

3. Determine the moment required, $M_{0\text{req}}$, and $M_{28\text{req}}$ to cause these stresses as

$$M_{0\text{req}} = \frac{\sigma_0 I}{y} \text{ and } M_{28\text{req}} = \frac{\sigma_{28} I}{y} \quad (\text{A.3})$$

where y is the distance to the outer fiber [half the pipe outside diameter of $OD = 6.9$ in./2 = 3.45 in. (87.6 mm).] The pipe internal diameter is $ID = OD - 2 t_w$ where t_w is the wall thickness = 0.245 in. (6.22 mm.) The moment of inertia, I , is

$$I = \frac{\pi}{64} (OD^4 - ID^4) = 28.4 \text{ in.}^4 = 1.18 \times 10^7 \text{ mm}^4 \quad (\text{A.4})$$

4. Determine the actuator forces required, $P_{0\text{req}}$ and $P_{28\text{req}}$ to achieve these moments. The moment in the four-point bending test at the pipe center is

$$M_0 = \left(\frac{P}{2}\right) \left(\frac{L}{3}\right) \quad (\text{A.5a})$$

and the moment at the ± 28 in. (± 711.2 mm) is

$$M_{28} = \left(\frac{P}{2}\right)\left(\frac{L}{6}\right) \quad (\text{A.5b})$$

Then the required actuator forces based on the moments at the pipe centerline and ± 28 in. (± 711.2 mm) are

$$P_{0\text{req}} = \frac{6M_{0\text{req}}}{L} \text{ and } P_{28\text{req}} = \frac{12M_{28\text{req}}}{L} \quad (\text{A.6})$$

where P is the applied actuator force and L is the total length 84 in. (213.4 cm.)

Figure A.2 shows the measured forces and the back-calculated forces using the method described above. The maximum measured actuator force is $P = 2.29$ kips (9.9 kN). The required forces are $P_{0\text{req}} = 3.90$ kips (17.3 kN) and $P_{28\text{req}} = 3.65$ kips (16.2 kN.) The required forces are roughly double the measured actuator forces. Figure A.3 compared back-calculated required forces based on centerline strains and those at the third-points. The two methods are in excellent agreement.

The theoretical deflections at the centerline and third-points for the geometry of this four-point bending test are calculated as shown in Figure A.4. Based on $P_{0\text{req}}$ the centerline deflection is $\Delta_{x=0} = 3.38$ in. (85.9 mm.) The two are in reasonable agreement considering the general methodology and the similarity between calculated values at the centerline and ± 28 in (± 711.2 mm) gage planes. Figure A.5 shows the required load based on strain gage reading at the centerline vs. centerline deflection from the centerline string pot and that calculated from beam theory. The two are in close agreement. If material nonlinearity was considered the back-calculated loads would be in closer agreement to the theoretical.

Figure A.6 shows a comparison of the measured actuator forces and calculated forces. The calculated loads are those based on crown and invert strain at the pipe centerline. These loads will be used for subsequent data analyses.

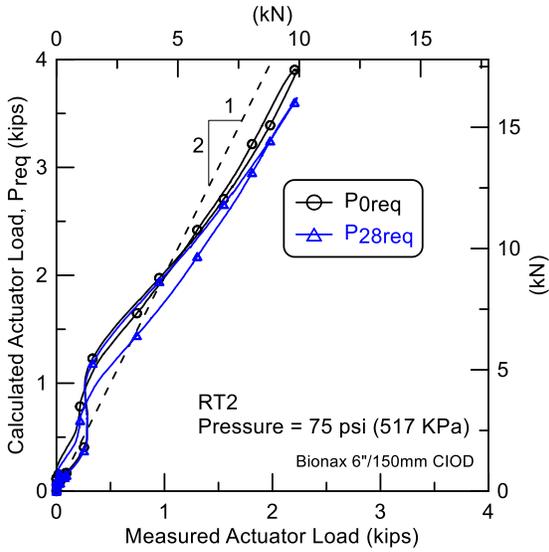


Figure A.2. RT2 Back-Calculated and Measured Actuator Loads

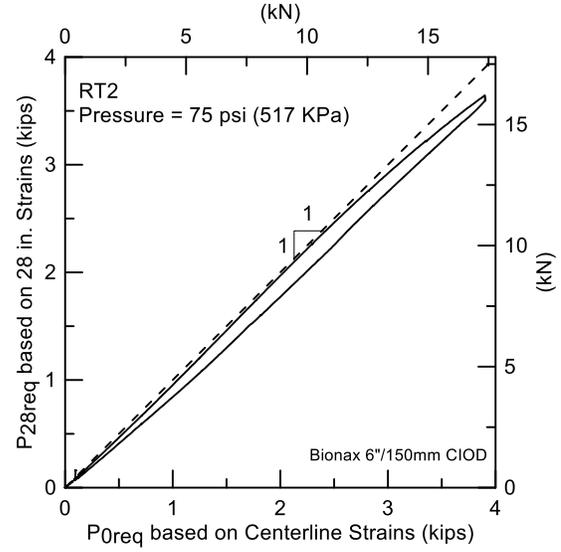
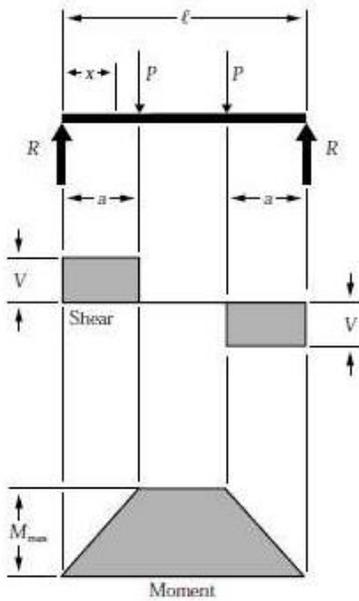


Figure A.3. RT2 Back-Calculated Loads based on Strains at Centerline and ±28 in. (±711.2 mm)



$$\begin{aligned}
 R &= V \dots \dots \dots = P \\
 M_{max} \text{ (between loads)} &\dots \dots \dots = Pa \\
 M_x \text{ (when } x < a) &\dots \dots \dots = Px \\
 \Delta_{max} \text{ (at center)} &\dots \dots \dots = \frac{Pa}{24EI} (3\ell^2 - 4a^2) \\
 \Delta_x \text{ (when } x < a) &\dots \dots \dots = \frac{Px}{6EI} (3\ell a - 3a^2 - x^2) \\
 \Delta_x \text{ (when } x > a \text{ and } < (\ell - a)) &\dots \dots \dots = \frac{Pa}{6EI} (3\ell x - 3x^2 - a^2)
 \end{aligned}$$

Figure A.4. Four-Point Bending Test Deflection Equations (after AISC, 2005)

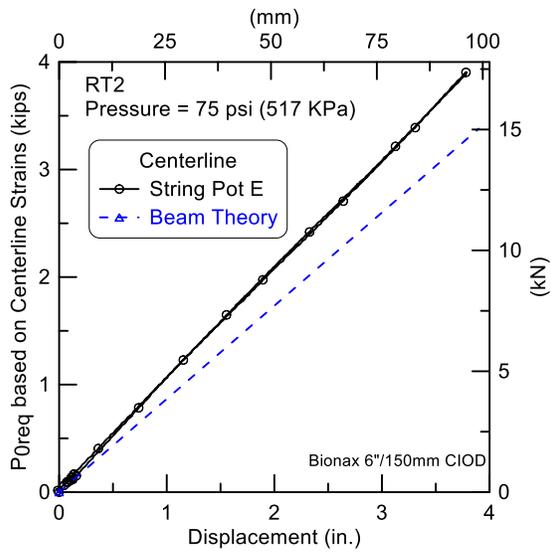


Figure A.5. RT2 Calculated Loads vs. Displacement at Pipe Centerline

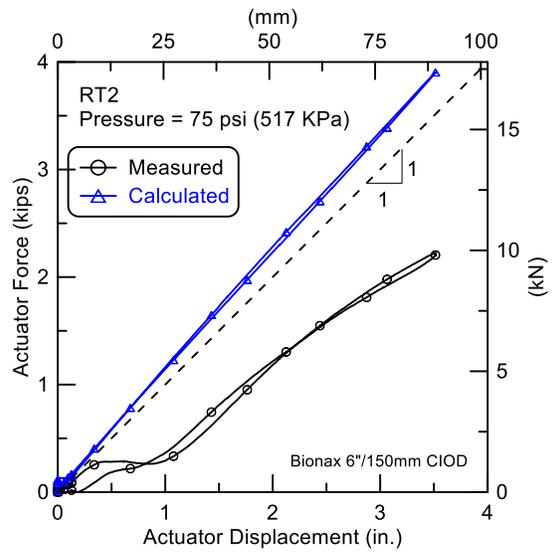


Figure A.6. RT2 Measured and Calculated Load vs. Actuator Displacement

Appendix B – Test Photographs



Photo B.1. Test RT2 Lab Setup

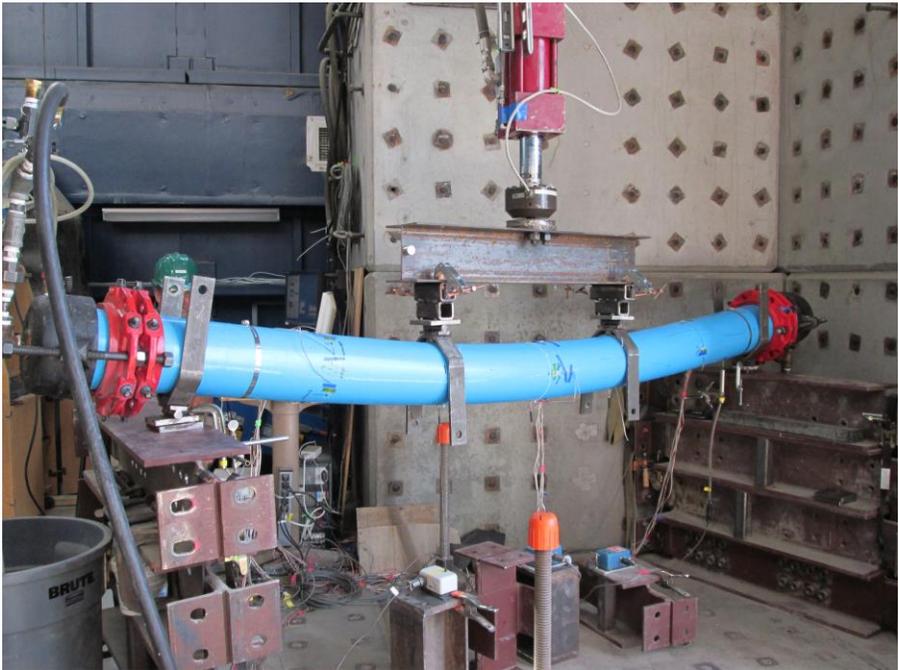


Photo B.2. Test RT2 Loading

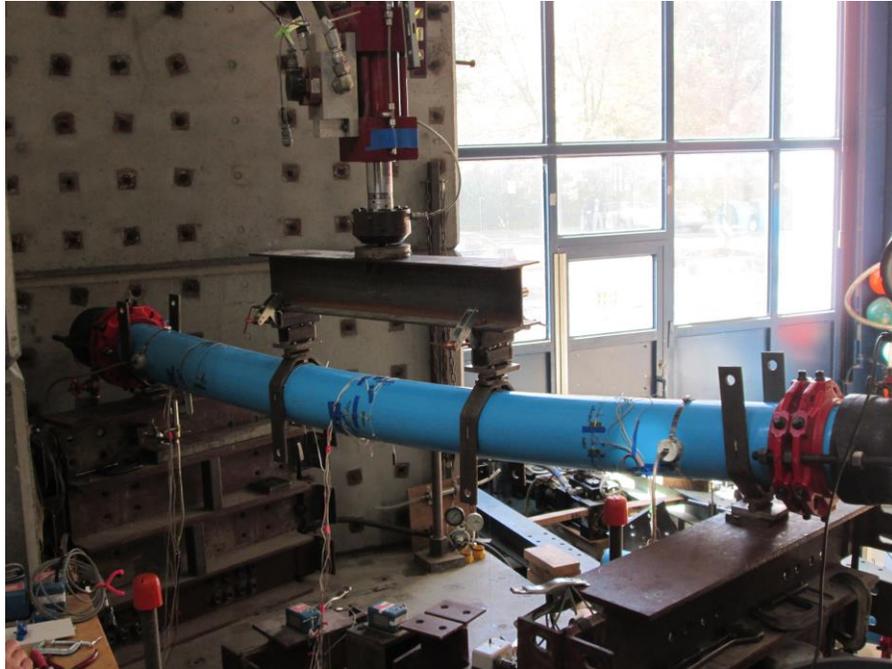


Photo B.3. Test RT2 during Bending



Photo B.4. Test Specimen RT3 Preparation



Photo B.5. Test RT3 Joint Instrumentation

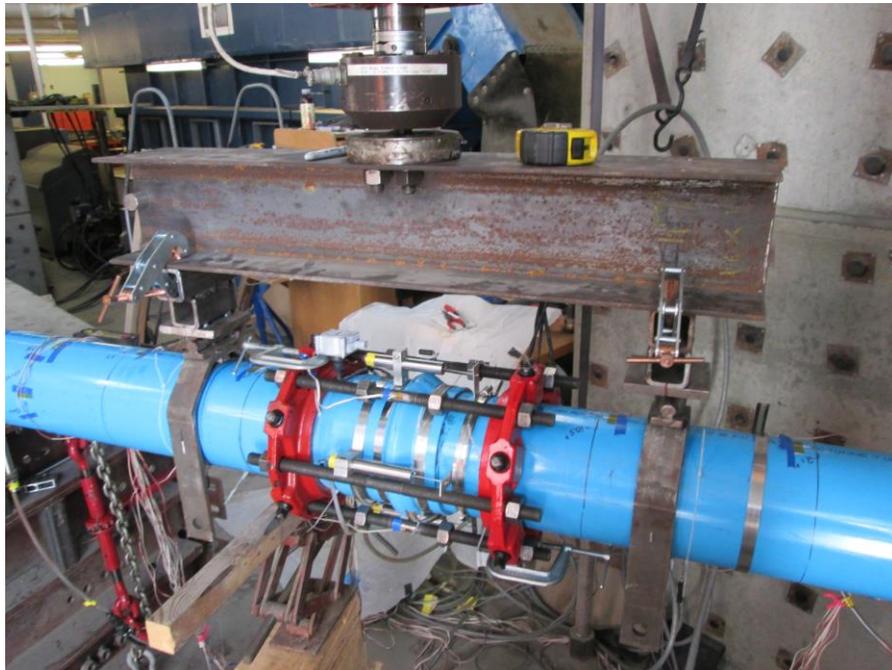


Photo B.6. Test RT3 Loading Configuration

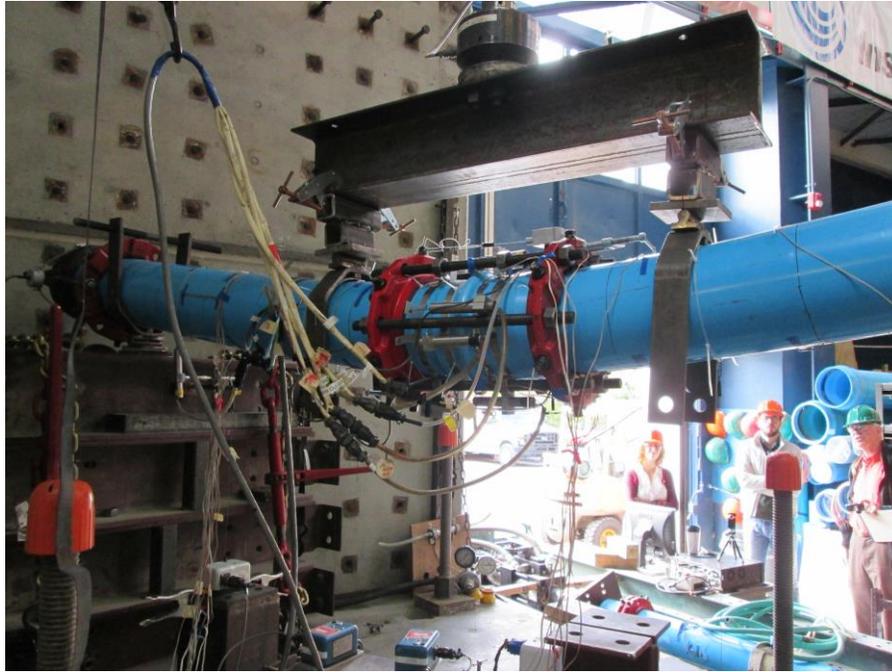


Photo B.7. Test RT3 during Bending