MURI presentation

Gennady Samorodnitsky

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I am working on:

- long memory models with heavy tails and connections with ergodic theory

- tail inference: multivariate and dynamic.

Mutlivariate tail estimation

A random vector $\mathbf{Z} = (Z^{(1)}, \dots, Z^{(d)})$ is regularily varying if there are functions $b^{(i)}(t) \to \infty$, $i = 1 \dots, d$, such that both

$$t\,\mathbb{P}\left[\left(\frac{Z^{(i)}}{b^{(i)}(t)}\right) \in \cdot\right] \xrightarrow{v} \nu_{\alpha_i},\ \ t \to \infty,\ \text{ on } (0,\infty],\ i = 1, \dots, d,$$

$$t\,\mathbb{P}\left[\left(\frac{Z^{(1)}}{b^{(1)}(t)}, \frac{Z^{(2)}}{b^{(2)}(t)}, \dots, \frac{Z^{(d)}}{b^{(d)}(t)}\right) \in \cdot\right] \xrightarrow{v} \nu, \quad t \to \infty$$

on $\mathbb{E} = (0,\infty]^d \setminus \{\mathbf{0}\}$. The problem: estimate the tail measure ν.

The problem can be transformed to the standard form.

Let F_i, $i = 1, \ldots, d$ be the marginal distribution functions. Define the generalized inverse function by

$$u_i(x) = \left(\frac{1}{1 - F_i} \right)^{\leftarrow} (x), \; i = 1, \ldots, d \,.$$

Then the marginals are standard Pareto, while

$$t \, \mathbb{P} \left[\left(\frac{u_i^{\leftarrow}(Z^{(i)})}{t}, \, i = 1, \ldots, d \right) \in \cdot \right] \xrightarrow{v} \nu_*(\cdot) \;\; \text{on } \mathbb{E}.$$

The problem: estimate the standard tail measure ν_*.

Issues:

- Use ranks
- What ranked observations belong to the tails?

Example The model is $X^{(i)} = |Z + Y^{(i)}|, i = 1, 2$ where $Z \sim N(0, 1)$ and $Y^{(1)}$ and $Y^{(2)}$ are independent identically distributed random variables independent of Z, with regularly varying tails.

Specifically: $Y^{(i)}$, $i = 1, 2$ have a Generalized Pareto distribution with parameters $\mu = 1, \alpha = 1, \sigma = 2$. The distribution function of such a distribution is given by

$$G_{\alpha,\sigma}(y) = 1 - \left(1 + \frac{1}{\alpha}\frac{(y-\mu)}{\sigma}\right)^{-\alpha}, \quad y \geq \mu\,.$$

n	Adaptive bias	Adaptive variance	5% bias	5% variance
1000	.0328	.0037	.0217	$3.886 * 10^{-4}$
5000	.0119	.0015	.0214	$7.889 * 10^{-5}$
20000	.0062	$5.282 * 10^{-4}$	.0214	$2.026 * 10^{-5}$

Table : Simulation results

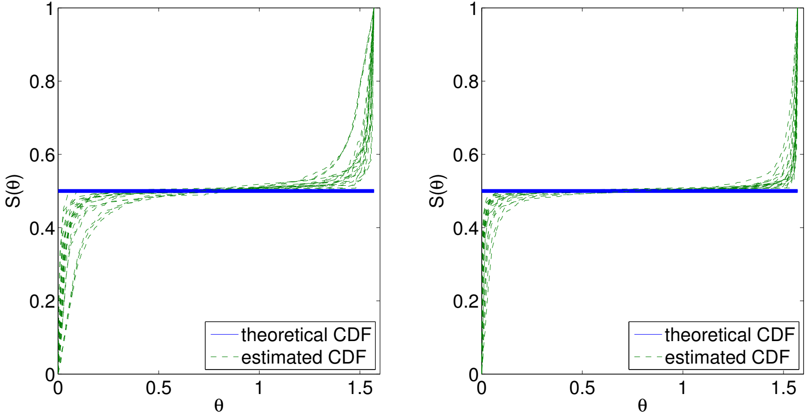

Figure : Estimated spectral measures from 20 simulations with $n = 5000$ and $n = 20000$

I planning to start working on tail changepoint detection

Given a time series $X_1, X_2, \ldots, X_n$ with some structural assumptions, decide:

- do at some point between 1 and n the tails change?
- if they do, when does it happen?