MURI presentation

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This year I have completed parts of the following projects:

- how multivariate heavy tails are affected by long memory;
- how does one estimate multivariate heavy tails;
- how dose one approximate ruin probabilities for heavy tailed systems?

I have also worked on:

- modelling explosions of IEDs with John Nolan, Edan Ben-Ari and others in Natick;
- multivariate heavy tails in social networks with Sid Resnick, Richard Davis and Don Towsley.

How multivariate heavy tails are affected by long memory.

Together with a former PhD student, Takashi Owada, supported by MURI, we have demonstrated how ergodic theory provides new insights into multivariate heavy tails.

Apparently very abstract tools of ergodic theory allow us to quantify the length of memory and ther effect on heavy tails.

Ergodic-theoretical tools also provde a very convenient and flexible procedure to build models with multivariate heavy tails.

How dose one approximate ruin probabilities for heavy tailed systems?

In either queuing applications and risk applications, estimating the ruin probability is of a crucial importance. Those are probabilities of the type

$$\psi(u) = P(\sup_{t \geq 0} X(t) > u)\,,$$

where $X(t)$ is either buffer content at time t, or claims total by the time t.

When the process $(X(t)$ has multivariate heavy tails, we (with B.-F. Nielsen and M. Bladt) provide a procedure to approximate the multivariate heavy tails of $(X(t)$ by a special type of multivariate heavy tails, those with infinite matrix-exponential distributions.

These allows us to derive efficient computational procedures to calculate the ruin probability.

Next year I am planning to do work on:

- multivariate heavy tails in social networks modeled as random graphs;
- inverse problems for multivariate heavy tails;
- negative dependence with multivariate heavy tails and resulting self-similar processes;

What part of my work could not have been accomplished without collaboration from the team members:

- With Sid Resnick we have been discussing understanding the joint heavy tails of the in-degree and the out-degree of certain random graphs.
- Sid has been discussing these issues with Richard Davis and Don Towsley.

Information on the degree distribution in a random graph is typically obtained either through a limiting procedure using as martingale-type argument, or through a direct heuristic argument.

In both cases one often obtains a recursion for the joint node degree probabilities.

It is typically hard to solve the recursion. One is usually able to calculate the marginal heavy tails of in-degree and the out-degree distribution, but not the joint tails, except in special cases.

We are working on developing a procedure that uses a new type of a multivariate Tauberian theorem to calculate the joint tail measure of the in- and out-degrees.

We took a recursion from a paper of Krapivsky et al., “Degree distribution of growing networks”.

$$\big[i + a(j + \mu) + b\big] p_{ij} = (i - 1 + \lambda) p_{i-1,j} + a(j - 1 + \mu) p_{i,j-1}$$

for $i \geq 0,\ j \geq 1,\ p_{01} = 1 + p\lambda$.

We have derived a PDE for the generating function of the (in-out)-degree distribution and solved it.

By using a new type of Tauberian theorems, we have proved the multivariate regular variation and obtained the tail measure.