Multivariate Subexponential Distributions

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1. Previous Definitions
2. Solvency Cones
3. Ruin Probabilities

Cline & Resnick, 1992

- $F \in \mathcal{L}(\nu; \mathbf{b})$ if $tF(\mathbf{b}(t) + \cdot) \xrightarrow{v} \nu$.
- $F \in \mathcal{S}(\nu; \mathbf{b})$ if $F \in \mathcal{L}(\nu; \mathbf{b})$ and $F_i \in \mathcal{S}$ for all i.
- Natural, works well with higher order convolutions, multivariate regular variation, and random sums, closed under marginal transformations.
- Hard to verify, does not work well with linear combinations of marginal distributions.

Cline & Resnick, 1992

$(X, Y) \notin \mathcal{S}(\nu; \mathbf{b}),\ X + Y \in \mathcal{S}$

$$P(X > x, Y > y) = \frac{1 + \gamma \sin(\log(1 + x + y)) \cos(\frac{1}{2}\pi \frac{x-y}{1+x+y})}{1 + x + y}.$$

We can show that $X + Y \in \mathcal{D} \bigcap \mathcal{L}$.

$(X, Y) \in \mathcal{S}(\nu; \mathbf{b}),\ X + Y \notin \mathcal{S}$

$P(X + Y = 2^n) = \frac{1}{2^{n+1}}$, for $n \geq 0$, with mass distributed uniformly on $\{(x, y) | x, y \geq 0, x + y = 2^n\}$ for each $n \geq 0$.

Omey, 2006

- $F \in S(\mathbb{R}^d)$ if $\lim_{t\to\infty} \frac{\overline{F^{(2)}}(t\mathbf{x})}{\overline{F}(t\mathbf{x})} = 2$ for all $\mathbf{x} > \mathbf{0}$.
- Only requires marginal distributions to be subexponential.

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Tails Defined by Cones

Let K be a cone such that

- $K \subset \{\mathbf{x} : \sum x_i \leq 0\}$.
- $\{\mathbf{x} : x_i \leq 0 \ \forall i\} \subset K$.

We consider tails of the form $u(\mathbf{b} + K)^c$, $\mathbf{b} > \mathbf{0}$.

- Define $F_{\mathbf{b},K}(u) = P(X \in u(\mathbf{b} + K))$.
- Take special note that $P(X_1 + X_2 \in u(\mathbf{b} + K)^c) \neq \overline{F^{(2)}_{\mathbf{b},K}}(u)$.

Reduction to One Dimension

- If $F_{\mathbf{b},K}(u)$ is subexponential, then

$$\lim_{t\to\infty} \frac{P(X^1 + \cdots + X^n \in t(\mathbf{b}+K)^c)}{P(X \in t(\mathbf{b}+K)^c)} = n.$$

- If X is multivariate regularly varying, then $F_{\mathbf{b},K}(u)$ is subexponential.

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Setting

- Let $X_i \in \mathbb{R}^d$ denote losses and $Y_i \in \mathbb{R}$ denote inter-arrival times.
- Assume initial buffer capital is $u\mathbf{b}$, premium vector is $\mathbf{p}$, and solvency cone is $-K$.
- In this case the probability of ruin is

$$P\left(\sum_{i=1}^{n}[X_i - \mathbf{p}\,Y_i] \in u(\mathbf{b}+K)^c \text{ for some } n\right)$$

Reduction to 1-dim Random Walk

- Define $W_n = \inf\{u \mid X_n \in u(\mathbf{b} + K)\}$.
- In this case $W_n \sim F_{\mathbf{b},K}$.
- We wish to reduce the d-dimensional random walk of step sizes $X_i - \mathbf{p}\,Y_i$ to a 1-dimensional random walk of step sizes $W_i - Y_i$.