MURI research

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Summary of work done in the last year with the emphasis on the last 6 months

In the last year I have been working on several topics.

1. Multivariate heavy tails in stochastic geometry.
2. Estimation algorithms in large networks (with Richard Davis and Jingjing Zou).
3. Detecting change in multivariate heavy tails (with Julian Sun).
4. Generating mechanisms for multivariate heavy tails (with the Amherst group, and Emily Fisher).
5. Degree growth for fixed nodes in a network (with Sid Resnick).

Multivariate heavy tails in stochastic geometry

- Stochastic geometry consists of studying the geometry of random objects in space.
- The objects may be a model of obstacles for movement or for communication.
- They may also be a model for impurities in materials.
- Heavy tails arise in the size of the random objects and in the parts of the space free from obstacles.

The simplest model: the Boolean spherical model. The centers form a Poisson process in space.

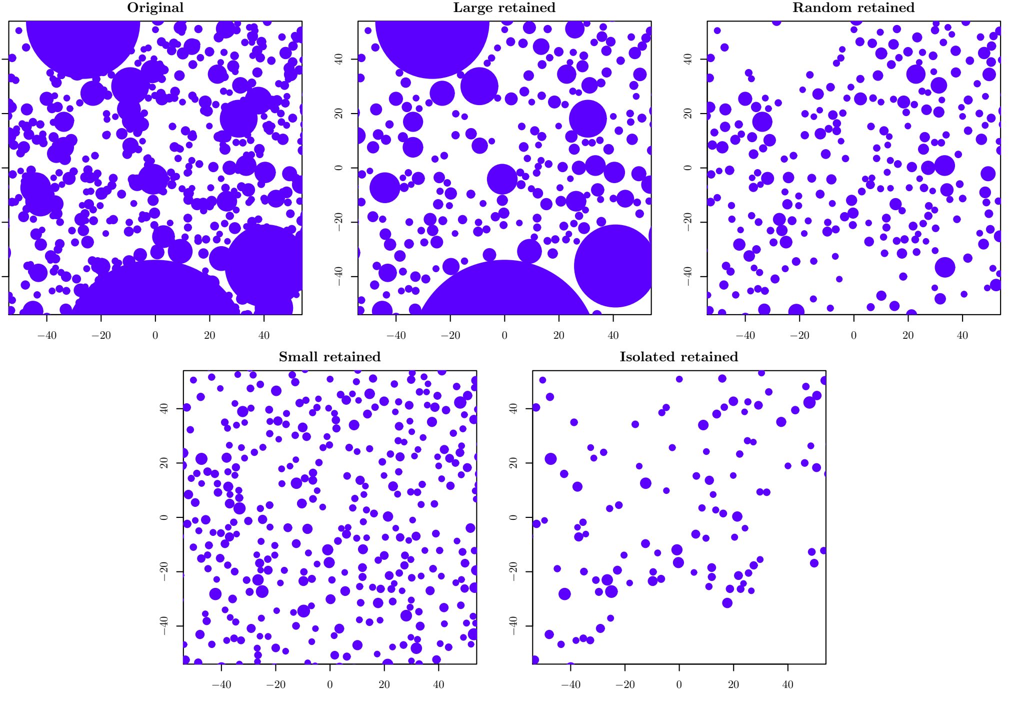
The radii of the spheres are i.i.d. These are assumed to be heavy tailed.

In this model some balls will overlap. This conflicts with some applications where a hard-core model is required.

A hard-core model is obtained by thinning. Thinning removes one or more balls in each overlap.

Different types of thinning:

- a large ball wins,
- a smaller ball wins,
- a random ball wins,
- only isolated balls stay.



Original
Large retained
Random retained
Small retained
Isolated retained
−40
−20
0
20
40

An important feature of the model: the contact distribution.

$$H(r) = P\Big(\text{the distance from 0 to the nearest remaining ball} \leq r\Big).$$

Question: is the contact distribution heavy tailed if the ball radii are heavy tailed?

Suppose $P(R > r) \approx r^{-\theta}$, $\theta > d$.

- If large balls win, then $\bar{H}(r) \approx r^{-2(\theta - d)}$.
- If only isolated balls remain, then $\bar{H}(r) \approx r^{-(\theta - d)}$.
- If random balls win, then $\bar{H}(r) \approx r^{-\theta}$.
- If small balls win, then $\bar{H}(r)$ decays exponentially fast.