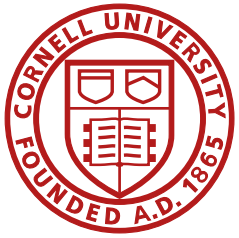

Changes in Extremal Behaviour

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Regular Variation Index

- A distribution F is regularly varying with index α if

$$\lim_{x\to\infty} \frac{\overline{F}(tx)}{\overline{F}(x)} = t^{-\alpha}.$$

- A common method for obtaining an estimator for α is using the Hill estimator

$$H_{n,k} = \frac{1}{k}\sum_{i=0}^{k-1} \log \frac{X_{n-i,n}}{X_{n-k,n}}.$$

- We use method from Nguyen and Samorodnitsky (2011) to determine the tail region and obtain an estimator for α.

Estimates for α

Tail index α for AIG 2008

	09.10	09.11	09.12	09.15	09.16	09.17	09.18	9.19	9.22	9.23
α	2.213	2.213	2.226	2.192	2.070	1.951	1.951	1.951	1.873	1.598

Tail index α for ALL 2008

	10.17	10.20	10.21	10.22	10.23	10.24	10.27	10.28	10.29	10.30
α	2.343	2.307	2.022	2.002	2.002	2.002	1.982	1.961	1.960	1.965

Problems:

- Multivariate case?
- Estimator from Nguyen and Samorodnitsky (2011) for iid data only.

Extremal Clustering

Take stationary sequence (X_i), for (u_n) satisfying $n\overline{F}(u_n) \to \tau$ for some $\tau > 0$, the exceedance process

$$N_n = \sum_{i=1}^{n} \epsilon_{i/n} I_{\{X_i > u_n\}} \to_d N = \sum_{i=1}^{\infty} \xi_i \epsilon_{\Gamma_i},$$

where (Γ_i) is a Poisson process with intensity $\theta\tau$, and (ξ_i) are the distributions of multiplicities of exeedances.

In this case, θ is the reciprocal of expected value of ξ_i, and it is called the extremal index.

Exceedance level u_n

The exceedance levels (u_n) play a large part in the estimators. But many estimators assume a pre-determined level u,

Block Estimator Fix threshold u, take k_n blocks of size r_n, let A_n denote the total number of exceedances of u, and B_n denote the number of blocks with exceedances of u. $\hat{\theta}_n = B_n/A_n$.

Estimator $\hat{\theta}_n$ is heavily influenced by the choices of u and k_n.

Estimates for θ, $k_n = log(n)^2$

Extremal index θ for AIG

	01.2005 - 12.2006	01.2007 - 03.2009	04.2009 - 12.2013
$u = 2.5\%$	0.462	0.236	0.247
$u = 5\%$	0.325	0.180	0.165
$u = 10\%$	0.193	0.111	0.098

Extremal index θ for ALL

	01.2005 - 12.2006	01.2007 - 03.2009	04.2009 - 12.2013
$u = 2.5\%$	0.311	0.227	0.493
$u = 5\%$	0.191	0.153	0.355
$u = 10\%$	0.201	0.115	0.106