Multivariate Heavy Tailed Phenomena: Modeling, Diagnostics and Applications

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Legs of the Stool

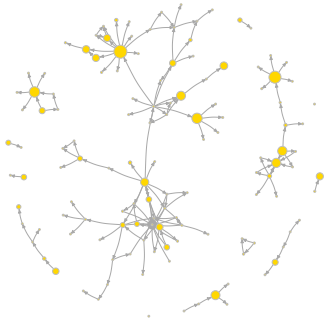
1. Mathematical modeling.
2. Data exploration, inference and network analysis.
3. Scheduling and control in multivariate heavy tailed environments.
4. Software.

1. Mathematical modeling

- Network growth under preferential attachment. (UMass, Columbia, Cornell)
- Generating mechanisms for multivariate heavy tails. (UMass, Cornell)
- Models for mobility, eg wireless networks. (OSU)
- Competition models under cumulative advantage; related to preferential attachment and generates games with heavy tailed durations. Cumulative advantage like preferential attachment may be linear or non-linear function of *fitness*. (UMass)
- Models to aid visual search. (UMass, Illinois)
  - Searching for suspects from a huge number of images.
  - Matching images when resolution is low in one image.
- Semi-parametric and asymptotic techniques for risk estimation. (Cornell)
- Methods for dimension reduction for multivariate heavy tailed data (ICA, PCA, ...). (Columbia, American)

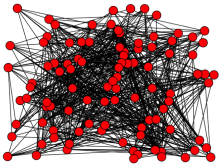
2. Data exploration, inference and network analysis.

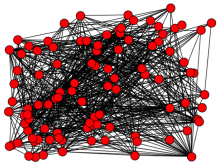
- Sampling issues: how to sample large networks under budget constraints to achieve estimation of summary quantities? (UMass, Columbia, Cornell, Minn)



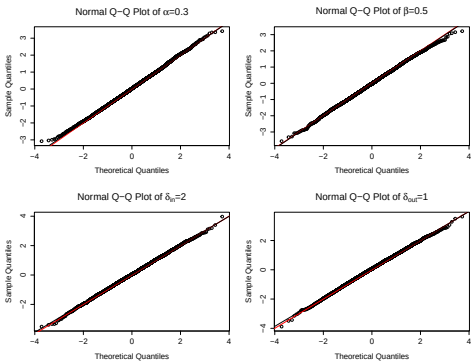
- Node classification: Questions are suggested by difficulties in sampling sensibly. Nodes may have multivariate characters: in- and out-degree, strengh of friendliness, strength of foe-y-ness, etc. (UMass, Cornell)

- Algorithms for matching large networks. (Illinois, UMass, Mn)
  - When are two networks similar?
  - Heavy tailed node degrees help the algorithm; match large degree nodes first.





- Calibration, simulation of a preferential attachment model. (UMass, Columbia, Cornell)
  - Efficiency loss from estimation using one snapshot vs network evolution;
  - stationary vs change-point analysis;
  - asymptotic semi-parametric methods (more robust) vs MLE parametric methods (more fragile but more accurate).

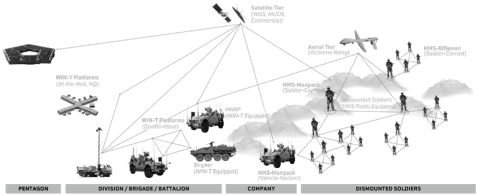


- Methods for exploring reciprocity (% of node pairs with bi-directional edges) in large directed graphs. How do reciprocal node pairs affect evolution and social structure? They form a core subgraph to which other nodes are attached. (Minn, UMass, Cornell)

- Mining geoMobility data (wireless users, bike-share, ambulance movement). Framework for study: EPIC. (Minn)
- Inference for heavy tail data (Columbia, Cornell)
  - Large values may be missing, lost, not-recorded.
  - Large values may be censored by rules. Must still do infence.
  - Limitations of minimum distance (Clausett) methods for threshold selection to decide what data has information about the tail.

3. Scheduling and control in multivariate heavy tailed environments.

- Emulating round robin scheduling in wireless networks. (Illinois)
  - Used for wireline where it is not sensitive to distributional properties of the file size distribution.
  - Military scenarios.
  - Goals to achieve: fairness, good throughput, small delay.



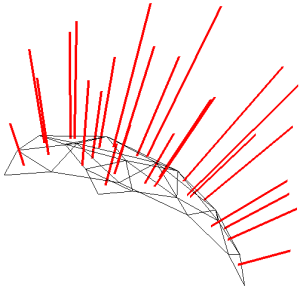
- Routing in cloud computing. (OSU, Illinois)
  - Massive data analysis is often needed to prepare for battle, to track adversaries, and for other military applications.
  - Routing algorithm that is insensitve to service time distribution.
  - Goals: Fast data retrieval, fast computation.
- Influence propagation in evolving networks. (Minn, OSU)

- Mobile data offloading: (OSU)
  - Cellular networks often highly constrained.
  - Suggests offloading some traffic to wifi or wired LANS.
- Context aware application scheduling for increasing battery lifetime and improving application response times in smartphones. (OSU)
  - Probability of launching an app follows Zipf's law.
  - The inter-run time and run time of an app follow multivariate heavy tail law.
  - Which apps held in memory and which should be closed?

4. Software.

Nolan (American) packages in R environment posted on CRAN (Comprehensive R Archive Network) at https://cran.r-project.org.

1. **mvmesh.** - a package to define and work with MultiVariate Meshes in n dimensions.
   - Define common shapes - hollow and solid spheres, simplices, rectangles, and tubes.
   - Plot these objects in 2 and 3 dimensions.
   - Define and manipulate shapes in higher dimensions.
   - Multivariate histogram functions
     - Count the number of data points in partitions of any of the above shapes,
     - Directional histograms that tally how many points are in a list of cones.

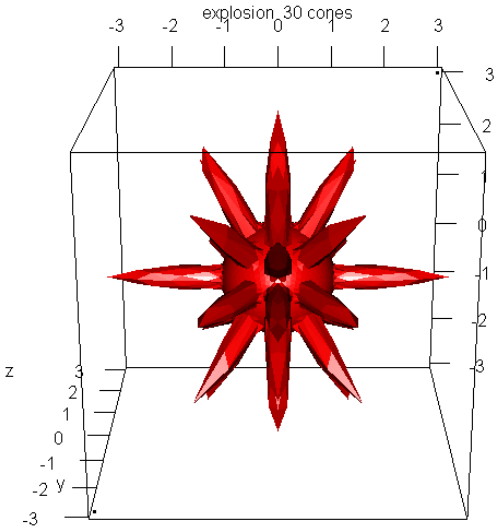


2. **SphericalCubature.** - a package to evaluate integrals over spheres in n dimensions. Gives exact formulas for polynomial integrands and adaptive methods for general functions.

3. **SimplicialCubature.** - a package to evaluate integrals over n dimensional simplices. Gives exact formulas for polynomial integrands and adaptive methods for general functions.

4. **gensphere.** - a package to define probability distributions that have level sets that are all scaled versions of a fixed contour.

   - Specify a large family of contours.
   - Define a probability distribution in terms of that contour and a radial decay function.
   - Arose out of previous Natick visit over attempts to model fragment dispersion from explosions.



4. **ecdfHT.** - a package for computing and plotting a transformed empirical cumulative distribution function (ecdf) for heavy tailed data.
    - Uses log-log transform with extremes pulled in to get visual diagnostic for heavy tails.
    - If plot suggests power law behavior, there are functions to estimate the exponent of the decay and the scale constant.
    - Multivariate generalizations are developed.
5. **mvevd.** (Under development:) Implements several dense classes of multivariate extreme value distributions.
    - discrete angular measure,
    - generalized logistic,
    - piecewise constant and linear angular measures.
    - Uses tools from the mvmesh and SimplicialCubature packages to work in dimension $n > 2$.

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