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Spatial/temporal coding for involuntary concept abstraction using linear dynamic systems

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***Abstract*— The main purpose of biological memory is to robustly recognize objects and concepts, not to reconstruct their copies. For this the memory should mostly record the relations among the constituents of the incoming signals, rather than the constituents themselves. Relational memories can be stored in the form of network connectivity between neurons. However, similarity testing for such spatial representations is difficult. We suggest that the spatial representations are converted to temporal representations for similarity testing, so as to enable a large scale approximate content addressable memory. This not only proposes a new way for automatic concept abstraction in data analysis, but also explains some of the mysteries in the behaviors of human and animal minds.**

I. Introduction

Steven Pinker calls the mind a system of organs of computation [16]. Rolf Landauer [12] noted that "Information is not a disembodied abstract entity; it is always tied to a physical representation. It is represented by engraving on a stone tablet, a spin, a charge, a hole in a punched card, a mark on paper, or some other equivalent." The information representations in the mind should be physical and computable. They should enable quick retrieval in a content addressable manner.

Neuroscience evidences suggest that neuronal appositions are densely and randomly distributed. This sets up a base for using neuronal spikes to construct connected neuronal networks as the spatial representations of signals. To enable efficient similarity testing the spatial representations should be converted to the temporal representations. Together these representations form a basis for a large scale content addressable memory which holds the key of many feats of human/animal intelligence [16], [15], [2]. In this paper we propose concrete algorithms for such conversions using linear dynamic systems. We outline the main thoughts in this section.

A biological memory system needs to record the gist of stimulation signals as experiences for similarity testing. In contrary to signal recordings in computer systems where the constituent components of the signals such as the image pixels and the audio waveform pieces are directly recorded, it is much more useful to record the relations among the raw components or component groups for checking the abstract similarities. A main function of a biological memory is recognition rather than reconstruction. A robust image recognition algorithm that is tolerant to small variations in shape, size and illumination would benefit more from the relational data in memory. We propose to convert spatial representations to temporal ones for computing the relations.

A simple algorithm for such a conversion is to use the linear dynamic system

$$\begin{aligned}\dot{\mathbf{x}}(t) &= S\mathbf{x}(t); \quad \mathbf{x}(0) = [1, 1, \cdots, 1];\\ y(t) &= [1, 1, \cdots, 1]\mathbf{x}(t)\end{aligned}$$

where the matrix S and the output $y(t)$ denote the spatial and temporal representations of the same signal, respectively. This has been proposed in [7], [8], [9] in the form of random walks over graphs and diffusions on manifold domains.

For very high dimensions, although the above conversion is theoretically accurate, it is numerically impractical. A natural approach is to divide the spatial representation S into small blocks and deal with each block separately. In terms of images since the relations among neighboring pixels are of more importance the matrix S is concentrated in the diagonal band. Furthermore, one can coarse-grain and compute only the relations among the neighboring pixel groups. Such relations can be enough for recognizing an image and the objects in it.

The concept of recognizing images and objects from the relations of the neighboring pixel groups explains our impressive visual sensing of the environment details. To be able to see so many objects so clearly means being able to detect very small changes in the retina signals of the environment. This can be achieved if enough relations among the image pixel groups are recorded and used for similarity testing.

We use simple images to explain what we mean by relations. An image is an $n \times n$ matrix I with elements I_{ij} denoting the parameters of the pixel at ij. If we restrict to grayscale images then I_{ij} is the intensity of the pixel, a real number, and I is a real matrix. We use the intensity differences and also the distances between pixels to make an adjacency matrix A, and calculate the graph Laplacian matrix L. When making the adjacency matrix we weight the differences between pixels with a decreasing function of their distances, such as the inverse of the distance square. When taking the difference of the pixel intensities if the

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difference is positive we use the value and if negative we give it a small positive weight for computational convenience. The adjacency matrix A as well as the corresponding graph Laplacian matrix $L = D - A$ where D is the degree matrix is asymmetric. In using the above algorithm to covert L into a time function we would like to see more oscillatory behavior in $y(t)$ so as to enhance the discriminating power. For this we split the matrix L into the summation of its symmetrical part and anti-symmetrical part as $L = 1/2(L + L^T) + 1/2(L - L^T)$. We then construct a new matrix $M = 1/2(L + L^T) + k/2(L - L^T)$ with the parameter k enhancing the skewness. Now we use this M in the place of the system matrix S in the above algorithm, and the resultant time function $y(t)$ is to be used for similarity testing. If we have two such spatial structures M_1 and M_2 such as two neighboring pixel groups in an image, we generate two time functions $y_1(t)$ and $y_2(t)$. For a simple implementation we can take a linear functional of the difference between $y_1(t)$ and $y_2(t)$ and the distance between the two pixel groups to form a relation between them. A more plausible version can be described as follows.

We start with how to convert a time function into a network/graph structure. This is analogous to writing a message to a piece of paper [12]. The paper here is a set of a huge numbers of second order dynamic systems often used to describe the inductance-capacitor-resistor (LCR) circuit. And the pen is the temporal function of concern. The LCR circuits are with diverse parameters such that each one of them responds more strongly to a particular behavior mode of the time function. This way the incoming time function picks up many waiting LCR circuits and make them connecting to each other following the neural plasticity rules to form a network of LCR circuits. One of the important features of the LCR circuits is resonance, which is the tendency to oscillate with greater amplitude at some frequencies than at others. Resonant systems are ubiquitous in Nature and it should be plausible for biology memory. The network formed in this manner will respond strongly when a similar time function comes again, and will in generally responds weakly to time functions that are different, accomplishing similarity testing in a involuntary manner.

When the two time functions $y_1(t)$ and $y_2(t)$ mentioned above hit the "paper" at the same time each of them will find some LCR circuits to produce strong currents. These currents in turn would make the connections between the circuits and form a network. The strength of the connections are determined by the two time functions and also the distances between the two pixel groups. If two neighboring pixel groups are very similar the connection between them will be strong and symmetric. If they are very different then the connection will be strong but directional.

At any stage of the memory system operation all existing LCR circuits, henceforth also referred as resonators, can be used as the "paper". This way many connections are made and an object or concept is defined by the connections of the representation network to other networks (of resonators). A macroscopic analogy is that a child encounters a new word when reading English texts. The child would try to register the word in mind by sound and then build up the understanding of the meaning based on the connections to neighboring words while reading further. It is these connections to other networks that define the meaning of a word or generally identify an object. In this process many resonators are repeatedly used but the connections to neighboring words/networks define the meaning. This enables different objects to share the constituent resonators and saves the storage space. Since resonator responses overlap in the frequency domain, the interferences generate noises which is especially harmful when information needs to be transmitted. To reduce this kind of errors one introduces gaps between the responding ranges of adjacent resonators (and we postulate this is the case in biological memory due to innate design). Such discretely spaced resonators responde more strongly to the mode close to the center of the spaced frequency range. The resonators are the basic units in a combinatoric scheme to describe a huge number of objects, from the very concrete images to the very abstract concepts. The resonators as the basic memory unit would be used repeatedly by different pieces of stimulation signals. The resonators having strong currents at the same time would establish strong connections. We assume that the information is coded by the numbers and the types of the resonators and the connection strengthes to the "neighboring" resonators, unless the coefficients are so small as to be negligible. We also note that the neuroscience rule of "spike timing dependent plasticity" (STDP) [14][1][4] dictates that the network thus formed is asymmetric.

Signals representing objects and concepts need to be transmitted for similarity testing in memory retrieval Signals representing an item consisted of multiple constituent pieces should be the aggregation of the "sub-signal representing each constituent piece. As such multiple-input single-output structure for signal aggregation and single-input multiple-output structure for signal branching are both needed. The signal operations should be essentially linear to utilize the properties of superposition, mode preservation and resonance.

Neuronal systems work in a warm and noisy environment. As mentioned before signal transmission in analogue settings cannot preserve high accuracy which is crucial, for example, in coding the retina signals in high accuracy in order to detect small changes. Our proposed spaced resonator system provides a level of digitalization, an idea widely used in computer and communications systems to battle the noises. Imaging a network of many resonators that are accessible via a random and dense connection fabric. The resonators' resonant frequencies are discretely spaced. These spaced resonators serve as quantization devices in the sense that the energy of a signal component is absorbed into the resonator with the closest resonance frequency. They also provide automatic recall with resonance. Furthermore they provide combinatorial power of representation similar to languages [16][17]. For example a possible scheme is to have symmetrically connected resonator groups as "letter"s and sequentially connected such groups as "words" and "sentences". Sequential connections are plausible with one-way STDP and symmetrical connection can be realized with STDP in both directions.

II. Network of resonators for memory

A. Resonators

Resonance is an ubiquitous phenomena. Parents pushing a swing set know they should push in phase with the swing set motion. A radio set selects the desired station by tuning the receiver circuit into the frequency that resonates with the station carrier wave. In a radio set receiver we set up an inductance-capacitor-resistor circuit (LCR circuit) so that the electrical and magnetic energies convert to each other with the frequency determined by the circuit parameters. In a swing set such frequency is determined by the swing height, similar to the pendulum in a grandfather clock. Neuronal circuits could perform as LCR circuits, with variable parameters. Although chemical/electrical dynamics would likely be the main mechanism for the LCR like behaviors, the topological arrangements of directional network links could also play a role. Some mathematical facts along this line will be discussed later.

A great feature of a resonant circuit is that it knows what signal component will get it really excited, in the way that a swing set knows how to select the pushing pattern to swing high. Now imagine that a mind is based on millions of such circuits with different groups of them connecting to different labels (which are also represented by resonator networks). If the query signal manages to reach vast many LCR circuits via a random connection fabric then the constituent query components would excite circuits representing similar components, and the corresponding label network would report the recognition of a concept, either concrete or abstract.

The resistor in the LCR circuit dissipates the energy into the environment to maintain stability and to reset. Unlike in the computers, energy dissipated in the surroundings in a brain is to be reused.

The transfer function of a resonator, or a second order band-pass filter, is

$$H(s) = \frac{H_0 \omega_0^2 s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \tag{1}$$

where $\omega_0 = F_0/2\pi$ is the frequency at which the gain of the filter peaks, $H_0 = H/Q$ is the circuit gain, and $Q = F_0/(F_h - F_L)$, with F_L and F_H as the frequencies where the response is -3dB from the peak, is the quality factor or the selectivity of the filter. When we choose high Q the pass band $F_H - F_L$ is pretty narrow. Note that $F_0 = \sqrt{F_H F_L}$ and the skirts of the response are symmetric around F_0 on a logarithmic scale. We assume that the "paper" in the brain consists of a large numbers of such filters with different ω_0s which are appropriately spaced in the range of concern (by innate design as a result of evolution).

B. Complex exponential convolution preserves the modes

The impulse response of an LCR circuit can be expressed as the sum of complex exponential functions referred as "modes". The complex exponential modes are preserved in the convolution operation. The importance of the temporal convolution is that when an input signal going through a linear dynamic system the output is the convolution of the input and the system impulse response function. For a linear system the input modes and the system's modes are preserved in the output, which is stronger when the two sets of modes are more similar. Such mode preservation property in addition to the spaced resonator network representation scheme preserves the signal modes when traveling through the paths and making connections to the sitting memories via excitation of resonators. The convolution of complex exponential functions $e^{-\lambda_k t}, k = 1, \cdots, n$ preserves all the modes in the sense that

$$e^{-\lambda_1 t} * e^{-\lambda_2 t} * \cdots * e^{-\lambda_n t} = c_1 e^{-\lambda_1 t} + \cdots + c_n e^{-\lambda_n t} \tag{2}$$

where $c_k = [(\lambda_1 - \lambda_k) \cdots (\lambda_{k-1} - \lambda_k)(\lambda_{k-1} - \lambda_k) \cdots (\lambda_n - \lambda_k)]^{-1}$ when all the λs are distinct, which covers all realistic cases. We note that this mode preservation property is unique to the linear dynamic systems and is important for coding and transmitting relational information. For example, an image could be divided into many small and overlapping blocks with each converted to a temporal representation consisted of complex exponential modes. The summations of portions of the block representation will retain the summand modes and try to find similar items in memory via mode resonance.

C. Resonator network representation of signals

The Fourier transform decomposes a time function as the sum of eternal sinusoids and and Laplace transform does this with eternal complex exponentials [13]. As such the uncertainty principle kicks in when representing signals. A compromise has to be made and we use impulse response function of second order linear dynamic systems, or resonators, as the basic constituents. These are the real parts of complex exponentials, or exponentially decayed sinusoids. They are causal functions but are not orthogonal to each other in terms of the usual inner product of time functions, which implies that the interferences among constituents would affect the information carried in the original signal. As we mentioned before, one natural approach is to introduce gaps between the neighboring basic constituents. In other words the frequency line is divided into alternating on and off intervals. For incoming signals with frequencies in an on interval there are resonators responding strongly to the incoming signal. And for the off interval frequencies there is no resonators responding with non-negligible output. This calls for high Q resonators.

Linear combinations of such resonators could approximate practical signals well. When a signal comes in it excites the resonators proportionally to its own decomposition weights since stronger components travel further. A particular component will excite strongly those that resonant with it in the sea of uniformly randomly distributed resonators, achieving a rough proportionality. The resonators get excited will excite other similar ones. This process may go on but the whole dynamic spreading process will end soon when other signals kick in and inhibit the current activities. The linear combination weights are not the only factor in accurately coding the relations. As mentioned before, the numbers and the types of the resonators that are used for this task play a more important role. Each resonator is like a colored bin and the combinations of colored bins, and the combinations of the combinations would make a memory scheme capable of storing millions of concepts. Note that the color here is the interval of a resonator and we have many of them. Also note that unlike the colors the resonators actively excites similar resonators once in action, providing the involuntary abstraction and consolidation.

The situation is a bit like coding thoughts using alphabets in languages. The combinatorial possibilities make language infinitely rich with only some thirty alphabet symbols. The sequential order of the alphabets in words, sentences and texts are important and it seems the resonator scheme described above does not have the sequential ordering mechanism. However the neuroscience finding of STDP actually provide a possibility here, in the sense that a group of resonators can be in a position to excite other groups and not be excite by them. Indeed STDP provides a mechanism to remember motions.

When a set of many resonators are simultaneously excited their output get aggregated due to hitting on synapses of the dendrite branches of the same neuron. The axon of this neuron then carry the sum of all the incoming signals and form a time signal to go somewhere. This time signal is capable to excite corresponding resonators elsewhere as long as it has that resonator component. This way a content addressable memory is working in multiple scales in space and in time.

Finally we note that once the connection to the sum neuron is formed we have a concept. Without this fixation the incoming signal is still just keeping wiring together the resonator components. Only when the sum neuron's touching points are fixed then the spatial representation of the incoming signal gets settled. This point is important in understanding the interactions of thinking and language. Language provides labels for the active resonator groups to settle in and narrow down to the stronger connections, although we may loose good ideas in the process.

D. Code the relations into spatial representation

We use visual image as an example to illustrate the coding process. An image sends photons to stimulate the photon sensors in the retina which turns the stimulations into electrical signals in the form of neuron spikes and send it to the visual cortex. Some processing occurs at this stage but our main concern is how the signals turn into spatial structure for memory so that if we see a similar image in a moment we can tell that this is similar to the one we just saw, even though we can hardly reconstruct the image when we close our eyes. This puzzling effect could be explained if the spatial structure recording the image is recording the relations among neighboring pixel groups. Because it is very hard to inverse the relations for reconstruction, while it is easy to test if the new image is similar to the one already coded via similarity testing of the relations. In general the more relations one codes the more accurate one can test the similarity because relations are constraints that the raw image signal has to satisfy. Relational coding also explains why we can record so many images one after another and often know which ones we have seen before. The same algorithm applies to audio signal recording as well. In other words we don't record the raw waveforms of the audio signal. Rather we record the relations of tiny blobs of the stimulating waveforms. This explains how can we carry a conversation with incomplete sentences and incomplete words. When enough recorded relations are testing positive the connected "meaning" is excited.

Relational coding is usually over complete in the sense that there are more relational constraints than necessary to uniquely determine the original signal. This is necessary for the accuracy in similarity testing when noise is abundant. In the case of visual image one generates relations among pixel groups and also relations of relations. The relational network generated for a single image is consisted of many subnetworks representing the objects and the relations among them. These subnetworks are connecting to other networks representing the names, colors and other labels. It is these connections that fix the network representations for objects.

III. Abstraction and analogy

With the above coding scheme in mind we proceed to discuss similarity testing for abstract concepts. Abstraction is the process of sifting the similarities from the instances or the differences among the instances. One of the impressive examples of abstract similarity testing is the psychology phenomena referred as analogical reminding [10], [17]. Analogical reminding is featured by the quickness, the abstractness, and the involuntariness of a memory recall carried out by the relatively slow neurons. The quickness calls for large scale content addressable memory. The abstractness needs the ability to sift commons from instances. And the involuntariness demands automatical recall. The spatial/temporal signal conversion described above could help achieving these. It emphasizes the use of the signal modes and spectral analysis. We use a simple example to illustrate the importance of the spectral information in the visually perceived similarity of spatial structures. Consider the images of independently generated white noises. These random graphs have similar spectral information, and are visually similar despite that they are constructed using independent random numbers and therefore mathematically independent.

To explain the idea of abstract similarity testing we restrict to the simpler case of of graph similarity testing scheme in [9]. The temporal representation of a graph/matrix is a vector that codes the spectral information of the matrix. Now we consider simple analogies in which one tests the similarity between the abstract relations that relate more concrete concepts. Consider two pairs of concepts (A, B) and (X, Y). Suppose it makes sense to say that "A to B is like X to Y", for example "a square A to a rectangle X is like a circle B to an ellipse Y". The following diagram shows the relational networks between the shape pairs. The link weights in the relational networks are determined by the temporal representations of the concepts.

$$\begin{array}{ccc} A & \xrightarrow{R_{AX}} & X \\ {\scriptstyle R_{AB}}\downarrow & & \downarrow{\scriptstyle R_{XY}} \\ B & \xrightarrow{R_{BY}} & Y \end{array}$$

Now the relational similarity statement that "A to B is like X to Y" can be understood as the similarity of matrices R_{AX} and R_{BY}, (and of R_{AB} and R_{XY},) which is mechanically the same as the similarity testing of the two concrete shapes.

The above similarity testing for relations does not need external triggers and leads to an involuntary experience, not unlike to recognizing facial expressions. Recognizing facial expressions such as a smile is crucial for human interactions. Consider two faces A and X, and their smile versions B and Y. The relational matrices R_{AB} and R_{XY} are similar and the similarity defines the concept of "smile". Human babies perhaps sift this out from smile faces early on. The relational network of smile could make connections to relevant concepts and it is possible that the silly smiles and sounds made by adults when holding a baby forms the neurological base for the sense of humor, which would be triggered by concrete or abstract silliness depends on the storage of abstractions in the mind.

The relational networks R_{AB} and R_{XY} are generated automatically (due to resonance among components of the temporal representations A, B, X, Y) and stored in the memory as part of the experiences. Such relational networks, and the relational networks for these relational networks, are all generated in subconscious and sitting there ready to be excited. This may help explaining the analogical reminding phenomena where a sequence of current events remind a possibly remote experience that is only similar in a very abstract manner.

IV. Motivating thoughts on spatial/temporal conversions

A. Conversion using diffusion in continuous domain

The idea of spatial/temporal signal conversion could be traced back to the famous 1966 paper by Mark Kac [11] where random walk analysis is used in the form of diffusion in a continuous domain. This work is followed intensively, see [18] and the references therein. For discrete domains like networks and graphs, an analogous equation for diffusion over a graph has been used in [3] for developing alternative page ranking algorithms in the Internet. The heat diffusion equation $\partial h_t/\partial t = -L_n h_t$ is a linear dynamic system in which the graph Laplacian L_n plays the role of the system dynamics matrix A in $\dot{x} = Ax + Bu$. In [7], [8], [9] we have been using $\partial h_t/\partial t = -L_n h_t$ and collecting the initial condition response for similarity testing. In these works it is generally assumed that the graph adjacency matrix is symmetrical. We highlight the intuitive ideas briefly below as they motivated our thoughts. In fact a discrete and asymmetric version with complex exponentials replacing the conductance would be a good model for the resonator networks discussed in this paper.

We consider an m-dimensional Riemannian manifold (M, g) and the associated Dirichlet Laplace-Beltrami operator $-\Delta_M$ acting in $L^2(M, dx)$, where dx is the volume measure on M induced by the metric g. Let $u : M \times [0, \infty) \to \mathbb{R}$ be the unique solution of

$$\frac{\partial u}{\partial t} = \Delta_M u, \quad t > 0,$$

with the initial condition

$$u(x, 0) = 1$$

and the boundary condition

$$u(x, t) = 0, \quad x \in \partial M, t > 0.$$

Using the spectral resolution $\{\lambda_k, \phi_k\}$ of Δ_M, where ϕ_k denotes the eigenfunction corresponding to the eigenvalue λ_k with $\lambda_1 < \lambda_2 < \cdots$, the Dirichlet heat kernel for M can be written as

$$p(x, y, t) = \sum_{k=1}^{\infty} e^{-\lambda_k t} \phi_k(x)\phi_k(y)$$

and the heat content in the domain is

$$\begin{aligned} h(t) &= \int_M u(x,t)dx = \int_M \int_M p(x,y,t)dydx \\ &= \sum_{k=1}^{\infty} e^{-t\lambda_k} \left(\int_M \phi_k(x)dx \right)^2 . \end{aligned}$$

We emphasize that $\int_M \phi_k(x)dx = \int_M \phi_k(x)u(x,0)dx$ is the Fourier coefficient of the initial condition $u(x, 0) = 1$ in the coordinate system $\{\phi_1, \phi_2, \cdots\}$. In other words the Fourier coefficients of the uniform initial condition as the reflection of the domain shape are carried by the exponential motion modes in the heat content to represent the spatial shape information. We also note that due to the unitary property of the Fourier transform the heat content coefficients are robust against small variations in the initial distribution.

In general if the operator involved in the diffusion type equations has distinct real eigenvalues, the time function $h(t) = \int_M u(x,t)dx$ is an exponential sum of the form $\sum_{i=1}^{\infty} \alpha_i e^{-\lambda_i t}$. Thus, in principle, one can extract the information about the non negligible parameters in $\alpha_i, \lambda_i, i = 1, \cdots, \infty$ from a small initial segment of $h(t)$. However it is worth noting that [6], [18] give examples where "isoheat" ($h(t)$ are the same) does not always imply isospetral (the eigenvalues are the same). These examples involve unusual constructions and should not concern us.

B. Complex eigenvalue implications in spatial structure

In most work on diffusion over graphs the graphs are assumed to be symmetric. A symmetric matrix has real eigenvalues and the diffusion over the associated graph behaves monotonically. However, an asymmetric graph is associated with complex eigenvalues that generate damped oscillatory behavior. To this end we list the following familiar mathematical facts that relates the spatial structural properties to the spectral features that would appear in the temporal representations. These facts have shed light on our thinking and may imply more understanding of the signal conversions discussed in this paper.

- The method of separation of variables is used to solve a wide range of linear partial differential equations with separable boundary conditions such as heat equation, wave equation, Laplace equation and Helmholtz equation. PDE solution via the separation of variables method shows that spatial and temporal expansion components often share the same frequency parameters. Thus, the time function obtained via integrating over spatial variables still preserves certain spatial structure information;
- The circle group is isomorphic to the special orthogonal group SO(2). This has the geometric interpretation that multiplication by a unit complex number is a proper rotation in the complex plane, and every such rotation is of this form. Since rotations generate sinusoidal time functions, this implies that a temporally oscillatory signal represents spatially circulating or spiraling structures. In particular, the spatial structural features of an asymmetric graph could be described by the curling densities over different regions and reflect in the temporal behavior of the dynamics of stuff moving in the graph;
- The connection between the spatial motion and complex eigen pairs (which can be easily turned into time functions) has been addressed by Helmholtz. He pointed out that the general motion of a sufficiently small non rigid body can be represented as the sum of (1) a translation (2) a rotation and (3) an expansion in mutually orthogonal directions. Note here that (2) corresponds to a skew-symmetric matrix and (3) a symmetric matrix. The same can be said about our eneryg/stuff, only that our stuff can do these actions with fractional quantities;
- [5] has shown that if E_n denotes the expected number of real eigenvalues of an $n \times n$ random matrix, then $lim_{n\to\infty} E_n/\sqrt{n} = \sqrt{2/\pi}$. This means the majority of the eigenvalues are complex. These complex eigenvalues are due to the fact that a randomly formed "graph" (with both positive and negative connections) would naturally have many small circular paths.
- A matrix A can always be decomposed as the sum of a symmetric matrix $S = \frac{1}{2}(A + A^T)$, whose eigenvalues are real, and a skew-symmetric matrix $R = \frac{1}{2}(A - A^T)$, whose eigenvalues are purely imaginary. The Lie product formula gives a physical picture of how stuff moves on the graph according to the linear dynamic equation $\dot{\mathbf{x}} = A\mathbf{x}$ since
$$e^{At} = e^{(S+R)t} = \lim_{n\to\infty} \left[e^{St/n} e^{Rt/n} \right]^n$$
shows that the stuff mixes the behavior of S and R, diffusing a bit, then rotating a bit, then diffusing a bit, then rotating a bit, and so on. This in turn provides a picture of stuff spiraling out on the graph following circularly expanding structures.

V. Concluding remarks

Dynamic systems are ubiquitous in nature, mostly for physical actions but also for signal conversions. In this paper we suggest that networks of spaced linear resonators can be used to perform conversions between the temporal and spatial representations of signals. While time functions are good for similarity testing, graph-like spatial structures are plausible for memory. Combined, they offer possibilities to explain certain functions in biological intelligence such as the quick recall of similar contents, as well as for the development of computer algorithms having similar functions.

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