Analysis of a Reciprocal Network using Google+: Structural Properties and Evolution

Braulio Dumba, Golshan Golnari, and Zhi-Li Zhang

University of Minnesota, Twin Cities, MN, USA,
{braulio,golnari,zhzhang}@cs.umn.edu

Abstract. Many online social networks such as Twitter, Google+, Flickr and Youtube are directed in nature, and have been shown to exhibit a nontrivial amount of reciprocity. Reciprocity is defined as the ratio of the number of reciprocal edges to the total number of edges in the network, and has been well studied in the literature. However, little attention is given to understand the connectivity or network form by the reciprocal edges themselves (reciprocal network), its structural properties, and how it evolves over time. In this paper, we bridge this gap by presenting a comprehensive measurement-based characterization of the connectivity among reciprocal edges in Google+ and their evolution over time, with the goal to gain insights into the structural properties of the reciprocal network. Our analysis shows that the reciprocal network of Google+ reveals some important user behavior patterns, which reflect how the social network was being adopted over time.

Keywords: Reciprocal Network ·Google+ ·Evolution ·Reciprocity

1 Introduction

Many online social networks are fundamentally *directed*: they consist of both *reciprocal* edges, i.e., edges that have already been linked back, and *parasocial* edges, i.e., edges have not been or is not linked back [1]. Reciprocity is defined as the ratio of the number of reciprocal edges to the total number of edges in the network. It has been shown that major online social networks (OSN) that are directed in nature, such as Twitter, Google+, Flickr and Youtube, all exhibit a nontrivial amount of reciprocity: for example, the global reciprocity of Flickr [2], Youtube [2], Twitter [3] and Google+[4] have been empirically measured to be 0.62, 0.79, 0.22 and 0.32, respectively. Reciprocity has been widely studied in the literature. For example, it has been used to compare and classify different directed networks, e.g., reciprocal or anti-reciprocal networks[5]. The authors in [1] investigate the factors that influence parasocial edges to become reciprocal ones. The problem of maximum achievable reciprocity in directed networks is formulated and studied in [6], with the goal to understand how bi-degree sequences (or resources or "social bandwidth") of users determines the reciprocity observed in real directed networks. The authors in [7] propose schemes to extract meaningful sub-communities from dense networks by considering the roles of users and their respective connections (reciprocal versus non-reciprocal ties). The authors in [8] examine the evolution of reciprocity and speculate that its evolution is affected by the hybrid nature of Google+, whereas the authors in [9] conduct a similar study and conclude that Google+ users reciprocated only a small fraction of their edges: this was often done by very low degree users with no or little activity.

Reciprocal edges represent the most stable type of connections or relations in directed network – they reflect strong ties between nodes or users [10–12], such as (mutual) friendships in an online social network or "following" each other in a social media network like Twitter. Connectivity among reciprocal edges can thus potentially reveal more information about users in such networks. For example, a clique formed by reciprocal edges suggest users involved are mutual friends or share common interests. More generally, it is believed that nontrivial patterns in the *reciprocal network* – the bidirectional subgraph (see Figure 1) of a directed graph could reveal possible mechanism of social, biological or different nature that systematically acts as organizing principles shaping the observed network topology [5]. Moreover, understanding the dynamic structural properties of the reciprocal network can provide us with additional information to characterize or compare directed networks that go beyond the classic reciprocity metric, a single static value currently used in many studies. However, little attention has been paid in the literature to understand the connectivity between reciprocal edges – the reciprocal network – and how it evolves over time.

In this paper we perform a comprehensive measurement-based characterization of the connectivity and evolution of reciprocal edges in Google+ (thereafter referred to as *G+* in short), in order to shed some light on the structural properties of G+'s reciprocal network. We are particularly interested in understanding how the reciprocal network of G+ evolves over time as new users (nodes) join the social network, and how reciprocal edges are created, e.g., whether they are formed mostly among extant nodes already in the system or by new nodes joining the network. For this, we employ a unique massive dataset collected in a previous study [9]. We start by providing a brief overview of G+ and a description of our dataset in Section 2. We then present our methodology to extract the reciprocal network of G+ using Breadth-First-Search (BFS), together with some notations in Section 3. In Section 4.1, we discuss a few key aggregate properties of the reciprocal network including the growth of the numbers of nodes and edges over time, the in-degree, out-degree, and reciprocal or mutual degree distributions. We then analyze the evolution of the reciprocal network in terms of its density, and categorize the nodes joining the reciprocal network based on the (observed) time they joined the network in Sections 4.2, and study the types of connections they make (reciprocal edges) in Section 4.3. Finally we discuss the implications of our findings and we conclude the paper in Section 5. We summarize the major findings of our study as follows:

– We find that the density of G+ – which reflects the overall *degree of social connections* among G+ users – decreases as the network evolves from its second to third year of existence. This finding differs from the observa-

tions reported in [8], where it found that G+ social density fluctuates in an increase-decrease fashion in three phases, but it reaches a steady increase in the last phase during its first year of existence.
- Furthermore, we observe that both the density and reciprocity metrics of G+'s reciprocal network also decrease over time. Our analysis reveals that these are due to the fact that the new users joining G+ later tend to be less "social" as they make fewer connections in general. In particular, i) the number of users creating at least one reciprocal edge is decreasing as the network evolves; ii) the new users joining the reciprocal network are creating fewer edges than the users in the previous generation.
- We show that if a user does not create a reciprocal edge when he/she joins G+, there is a lower chance that he/she will create one later. In addition, users who already have reciprocal connections with some users tend to create more reciprocal connections with additional users.

To the best of our knowledge, our study is the first study on the properties and evolution of a "reciprocal network" extracted from a *directed* social graph.

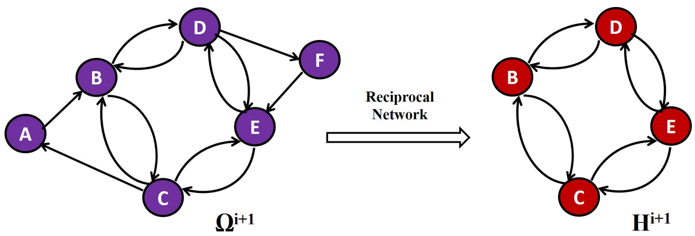


Fig. 1. Illustration of the reciprocal network (H^{i+1}) of a directed graph (Ω^{i+1}). Specifically, (B, C), (C, B), (B, D), (D, B), (D, E), (E, D), (C, E), (E, C) are reciprocal edges; (A, B), (C, A), (D, F), (F, E) are parasocial edges. The reciprocity of Ω^{i+1} is $8/12 = 0.67$

2 Google+ Overview and Dataset

In this section, we briefly describe key features of the Google+ service and a summary of our dataset.

Platform Description:June 2011 Google launched its own social networking service called Google+ (G+). The platform was announced as a new generation of social network. Previous works in the literature [8, 9] claim that G+ cannot be classified as particularly asymmetric (Twitter-like), but it is also not as symmetric (Facebook-like) because G+ features have some similarity to both Facebook and Twitter. Therefore, they labelled G+ as a hybrid online social network[8]. Similar to Twitter (and different from Facebook) the relationships in G+ are unidirectional. In graph-theoretical terms, if user[1] x follows user y this relationship can be represented as a directed social edge (x, y); if user y also has a directed social edge (y,x), the relationship x, y is called symmetric[13]. Similar to Facebook, each user has a stream, where any activity performed by the user appears (like the Facebook wall). For more informations about the features of G+ the reader is referred to [14, 15].

Dataset: we obtained our dataset from an earlier study on G+ [9], so no proprietary right can be claimed. The dataset is a collection of 12 directed graphs of the social links of the users[2] in *G+*, collected from August, 2012 to June, 2013. We used BFS to extract the Largest Weakly Connected Component(LWCC) from all of our snapshots of G+. We label these set of LWCCs as subgraphs Ω^i (for $i = 1, ..., 12$). Since LWCC users form the most important component of G+ network[9], we extract the reciprocal network of G+ from the Ω^i subgraphs (see Sect. 3). However, for consistency in our analysis, we removed from the subgraphs $\Omega^{i=1,...11}$ those nodes that do not appear in our last snapshot at Ω^{12}. Table 1 summarizes the main characteristics of the extracted Ω^i.

Table 1. Main characteristics of G+ dataset

ID	# nodes	# edges	Start-Date	Duration
Ω_1	66,237,724	1,291,890,737	24-Aug-12	17
Ω_2	69,454,116	1,345,797,560	10-Sept-12	11
Ω_3	71,308,308	1,376,350,508	21-Sept-12	13
Ω_4	73,146,149	1,406,353,479	04-Oct-12	15
Ω_5	76,438,791	1,442,504,499	19-Oct-12	14
Ω_6	84,789,166	1,633,199,823	02-Nov-12	35
Ω_7	90,004,753	1,716,223,015	07-Dec-12	40
Ω_8	101,931,411	1,893,641,818	16-Jan-13	40
Ω_9	114,216,757	2,078,888,623	25-Feb-13	35
Ω_{10}	125,773,639	2,253,413,103	01-Apr-13	25
Ω_{11}	132,983,313	2,356,107,044	26-Apr-13	55
Ω_{12}	145,478,563	2,548,275,802	20-Jun-13	N/A

3 Methodology & Basic Notations

In this section, we describe our methodology to extract the reciprocal network of G+. To derive the reciprocal network of G+, we proceed as follows: we extract the subgraphs composed of nodes with at least one reciprocal edge for each of the snapshots of Ω^i. We label these new subgraphs G^i (for $i = 1, 2, ..., 12$). By comparing the set of nodes and edges in each of the sugbraphs G^i, we observe

[1] In this paper we use the terms "user" and "node" interchangeable

[2] G+ assigns each user a 21-digit integer ID, where the highest order digit is always 1 (e.g., 100000000006155622736)

that a very small percentage of nodes depart G^i as it evolves (*unfollowing behaviour*[16]). Therefore, for consistency in our analysis, we removed from the subgraphs $G^{i=1,\dots 11}$ those nodes that don't appear in our last snapshot at G^{12}. We label these new set of subgraphs L^i (for $i = 1, 2, ..., 12$). However, L^i is not a connected subgraph. Hence, we use BFS to extract the Largest Weakly Connected Component (LWCC) for each of the snapshots of $L^{i=1,..12}$. We label these extracted LWCCs as subgraphs H^i (for $i = 1, 2, ..., 12$).

In this paper, we consider subgraph H^i as the "reciprocal network" of G+ [3]. In the next sections, we will focus our analysis on the structural properties and evolution of H^i. To achieve this, we extract subgraphs H^i_j composed of the set of users that join the network at snapshot i and j represents this subgraph at specific snapshots ($j => i$).

Let ΔH^{i+1} denote the subgraph composed with the set of nodes that join subgraph H^i_j at snapshot $j = i + 1$. Then, we define the following relationship (see Fig. 2):

$$H^{i+1} = H^i \cup \Delta H^{i+1} \tag{1}$$

In the following sections, we use subgraphs ΔH^{i+1}, H^i_j and (1) to analyse the reciprocal network of G+. For clarity of notation, we sometimes drop the superscript i and subscript j from the above notations, unless we are referring to specific snapshots or subgraphs.

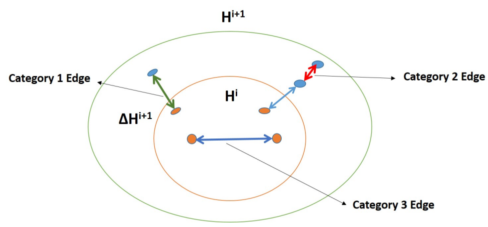


Fig. 2. Illustration of the relationship between subgraphs ΔH^{i+1}, H^{i+1}, H^i and the categories of the edges in subgraph H^i (for i=1,...,12)

4 Reciprocal Network Characteristics & Its Evolution

In this section, we present a comprehensive characterization of the connectivity and evolution of the reciprocal edges in G+, in order to shed an insightful light on the structural properties of the reciprocal network of G+. To achieve this, we proceed as follows: a) we provide a brief overview of the structural properties of the reciprocal network; b) we analyse the evolution of the density of the

[3] It contains more than 90% of the nodes with at least one reciprocal edge in G+. Hence, our analysis of the dataset is eventually approximate.

reciprocal network and c) we categorize the nodes joining the reciprocal network and their edges respectively.

4.1 Overview of the Reciprocal Network

We start by providing a brief overview of some global structural properties of the reciprocal network of G+, more precisely, the growth of its number of nodes and edges, as well as, its degree distributions:

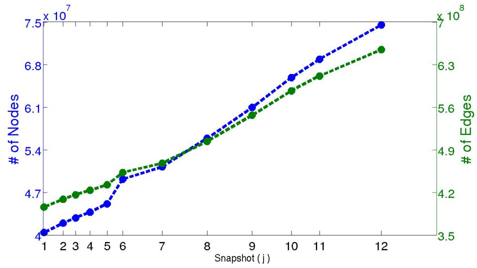
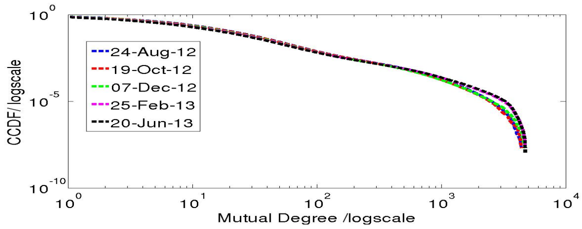


Fig. 3. Growth in the number of nodes and edges in H

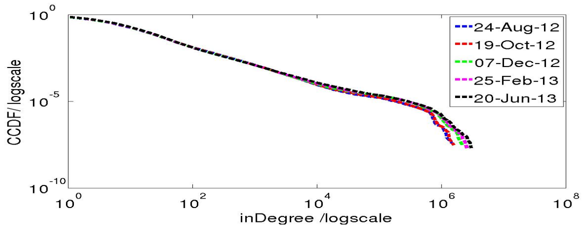
Nodes and Edges: Figure 3 plots the number of nodes (left axis) and edges (right axis) across time. We observe that the number of nodes and edges increase (almost) linearly as H^i evolves. The only exception is between H^i snapshots 5-6 ($19.Oct.12 - 02.Nov.12$), where we observe a significant increase in the number of nodes and edges. The time of this event correlates with the addition of a new G+ feature, on $31.Oct.12$, that allows users to share contents created and stored in Google Drive[17] directly into the G+ stream, as reported in[17]: "share the stuff you create and store in Google Drive, and people will be able to flip through presentations, open PDFs, play videos and more, directly in the G+ stream". Our dataset shows the impact of this event in G+: *it attracts more users to join G+ and many of these users might have already been using Google Drive in the past.*

In-degree, Out-degree and Mutual Degree Distributions: Figure 4 shows the CCDF for mutual degree, in-degree and out-degree for nodes in subgraphs H^i. We can see that these curves have approximately the shape of a Power Law distribution. The CCDF of a Power Law distribution is given by $Cx^{-\alpha}$ and $x, \alpha, C > 0$. By using the tool in [18, 19], we estimated the exponent α that best models our distributions. We obtained $\alpha = 2.72$ for mutual degree, $\alpha = 2.41$ for out-degree and $\alpha = 2.03$ for in-degree. We observe that the mutual degree and out-degree distributions have similar x-axis range and the out-degree curve drops sharply around 5000. We conjecture this is because G+ maintains a policy that allows only some special users to add more than 5000 friends to their circles [4].

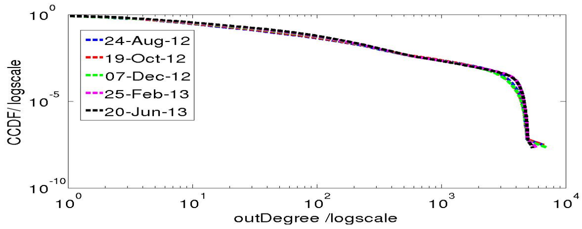
The observed power law trend in the distributions implies that a small fraction of users have disproportionately large number of connections, while most



(a) Mutual degree distribuition



(b) In-degree distribution



(c) Out-degree distribution

Fig. 4. Degree distributions for subgraph H^i

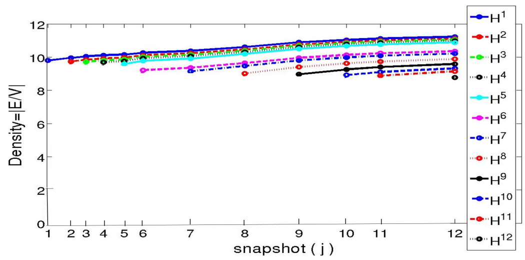
users have a small number of connections - *this is characteristics for many social networks*. We also observe that the shape of the distributions have initially evolved as the number of users with larger degree appeared.

4.2 Density Evolution & Nodes Categories

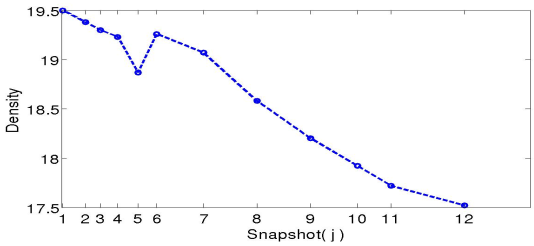
In this section, we analyze the evolution of the reciprocal network in terms of its density, and categorize the nodes joining the reciprocal network based on the (observed) time they joined the network. Next, we present our analysis:
Density: Figure 5(a) shows the evolution of the density of subgraph H^i, measured as the ratio of links-to-nodes[4]. We observe that as subgraph $H^{i=1,..12}$ evolves its density decreases. However, if we fix the number of nodes for each of the snapshots of H^i and analyse their evolution, we observe that the density is increasing (see Fig. 5(a)). From these results, we conclude that the new users

[4] We follow the terminology in [22] in order to compare with previous results

(ΔH^{i+1}) joining subgraphs H^i are responsible for the observed decrease in the density. Because these users initially create few connection when they join H^i (*cold start phenomenon*). However, the longer these users stay in the network, they discover more of their friends and consequently they increase their number of connections (edges). From the slopes of the graphs in Fig. 5(a), we observe that the new users are creating fewer links than the new users in the previous generation. Here, we define "previous generation" as the set of new users in the anterior snapshot, for example: the previous generation for new users in ΔH^3 are the users in ΔH^2.



(a) Density evolution - H^i



(b) Density evolution - Ω

Fig. 5. Evolution of the Density for graphs Ω and H

We also observe that the percentage of total users with at least one reciprocal edges in G+ decreases from 66.7% to 54.1% as the network evolves. Consequently, in our analysis, we also observe that the global reciprocity of G+ decreases (almost) linearly from 33.9% to 25.9%. From these results, we extract some important points: *a) the number of users creating at least one reciprocal edge is decreasing as the network evolves and b) the new users joining the reciprocal network are creating fewer edges than the users in the previous generation. Thus, the new users in G+ are becoming less social.*

Previous studies on social networks show that the social density for Facebook[20] and affiliation networks[21] increases over time. However, it fluctuates on Flickr[22] and is almost constant on email networks[23]. Differently, our dataset shows that the social density of G+ and of its reciprocal network (Fig. 5(a) and Fig. 5(b)) decrease as the network evolves. This is an interesting observation because it contradicts the *densification power law*, which states that real networks tend to densify as they grow[24].

The authors in [8] analysed the evolution of the social density of G+ using a dataset collected in the first year of its existence ($06.Jun.11 - 11.Oct.11$). They reported that G+ social density fluctuates in an increase-decrease fashion in three phases, but it reaches a steady increase in the last phase[8]. *Differently, our results shows that the social density of G+ is decreasing as the network evolves from its second to third year of existence* – the only exception is between snapshots 5 to 6, due to the events discussed in Sect. 4.1.

Node Categories: we classify the nodes joining H into the following categories (for clarity of notations we drop the superscript i and subscript j):

- Ω: node "x" exists in subgraph Ω at snapshot $j-1$ and joins H at snapshot j
- G: node "x" exists in subgraph G at snapshot $j-1$ and joins H at snapshot j
- L: node "x" exists in subgraph L at snapshot $j-1$ and joins H at snapshot j
- $NewArrival$: node "x" does not exist in the system at snapshot $j-1$ and joins both Ω and H at snapshot j

Figure 6(a) shows the distribution of the nodes joining H by categories. We observe that on average 63% of the nodes joining the subgraph H are new users in the system, 29% comes from the subgraph Ω and the remaining percentage comes from either subgraphs G or L. From these results we infer the following: *a) the majority of users that are joining the reciprocal network of G+ are new users in the system; b) if a user doesn't create a reciprocal edge when he/she joins G+, it is very unlikely that he/she will ever reciprocate a link in the network.*

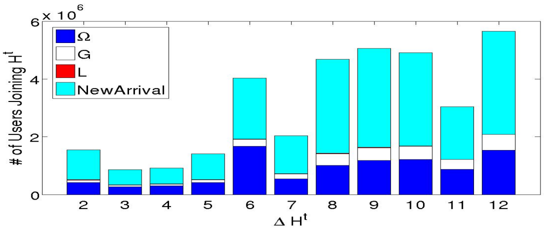
4.3 Edge Categories & Its Evolution

In order to understand the connectivity between the nodes in the reciprocal network, we analyse the evolution of the reciprocal edges in H^i. To achieve this, we restrict our analysis[5] to the subgraphs H^1 and H^2. Firstly, we present our edges categories. Secondly, we analyse the evolution of the degree distribution for each edge category:

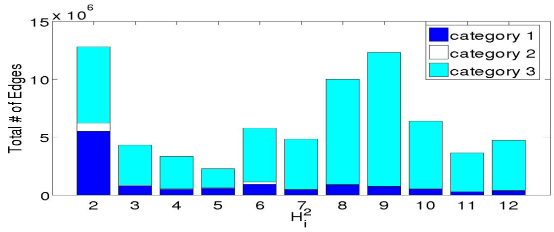
Edges Categories: we classify the edges created by nodes joining H^i into the following three categories (see Fig. 2 for an illustration):

- Category 1: $e(u,v)$ such that $u \in \Delta H^{i+1}$ and $v \in H^i$
- Category 2: $e(u,v)$ such that $u \in \Delta H^{i+1}$ and $v \in \Delta H^{i+1}$ and $\exists v^* \in H^i : e^*(u,v^*)$
- Category 3: $e(u,v)$ such that $u \in H^i$ and $v \in H^i$

[5] Similar results are obtained using the other subgraphs ($H^{i=3,\ldots,12}$)



(a) Total number of nodes joining H^i per category



(b) Total number of new edges per category created in H_j^2 for each of j snapshots

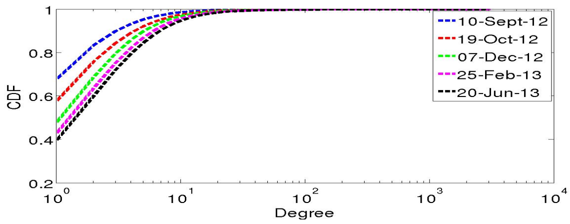
Fig. 6. Nodes and edges categories for subgraph H

Figure 6(b) shows the distribution of the edges based on the defined categories. We observe that most of the new edges seen across all snapshots of H_j^2 are due to category 3 edges. Furthermore, by looking at the last snapshot of H^i (for $i = 12$), we observe that 69% of the edges in H_{12}^{12} are between nodes in H^1 only. This result shows that although the density decreases as subgraph H^i evolves, the connectivity of a subset of its nodes is increasing (*densification*) and their connectivity accounts for a huge percentage of the total edges in the system.

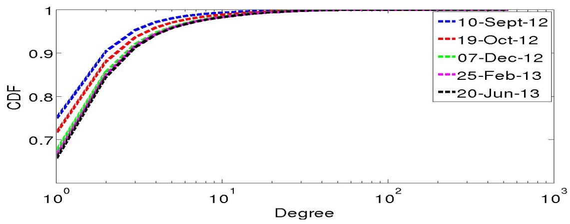
Degree Distribution: Figure 7 shows the degree distribution for all categories of edges and how they evolve across time. Figure 7(a) shows the CDF of the degree distribution for category 1 edges. From this figure, we observe that when new nodes (ΔH^2) join H_2^1, initially they create few connections, but the longer they stay in the system the number of connections to nodes already in the system increases significantly (as stated in Sect. 4.2). Furthermore, from our dataset, we observe that 72% of the nodes in ΔH^2 have only connections (edges) to nodes already in the system (H_2^1).

Figure 7(b) shows the CDF for the degree distribution of category 2 edges. From the results of Fig. 7(a) and Fig. 7(b), we infer that when new nodes (ΔH^2) join H_2^1, they create more connections with the nodes already in the system. Figure 7(c) shows the degree distribution for edges of category 3. We observe that the shape of the degree distribution is decreasing which implies that the network is become more dense (*densification*), as discussed above.

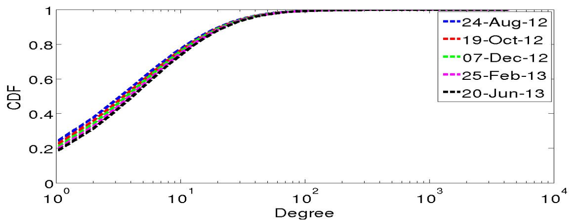
In summary, our analysis on the categories of nodes and edges yields the following key findings: *a) the majority of users that joins the reciprocal network* *of G+ are new users in the network and they tend to create reciprocal connections mostly to users who already have reciprocal connections to others; b) if a user does not create a reciprocal edge when he/she joins G+, there is a lower chance that he/she will create one later.*



(a) Category 1



(b) Category 2



(c) Category 3

Fig. 7. Degree distribution per edge category

5 Implication of our Results for G+ & Conclusion

In this paper, we present the first study on the properties and evolution of a "reciprocal network", using a massive G+ dataset. Analyzing the connectivity of reciprocal edges is important because they are the most stable type of connections in directed network and they represent the strongest ties between nodes: users with large number of mutual edges are less likely to depart from the network and they may form the most relevant community structure[6](the intimacy

[6] We will analyse the community structure in a reciprocal network as future work

community [7]) in directed OSN networks. Our analysis show that the reciprocal network of G+ reveals some important patterns of the user's behavior, for example: new users joining G+ are becoming less social as the network involves and they tend to create reciprocal connections mostly to users who already have reciprocal connections to others. Understanding these behaviors is important because they expose insightful information about how the social network is being adopted.

The findings here also provide hints that can help explain why G+ has so far failed to compete with Twitter and Facebook, as recently reported [25]. Firstly, we observe that although the numbers of nodes and edges increase as G+ evolves, the density of the network is decreasing. This result supports the claim that some users joined G+ because they need to access some of Google products but they weren't interested in creating connections in the network, in contrast to users in Twitter. Secondly, we observe a decrease in the reciprocity of G+ because the percentage of users with at least a reciprocal edge decrease as the network evolved. Furthermore, the users that joined the reciprocal network later always create fewer connections than the users who joined earlier. From this result, we infer that many users do not use G+ to connect and chat with friends, in contrast to users in Facebook[7]. Therefore, in its second year of existence, the G+ social network was already showing "signs" that it was failing to compete with others online social network, such as Twitter and Facebook. Many of the studies in the literature about G+ [4], [8, 9], [13] were done using dataset mostly collected in the first year of G+ existence. Thus, they either did not observe or failed to see these signs.

Our work is only a first step towards exploring the connectivity of reciprocal edges in social and other complex networks – reciprocal networks. There are several interesting directions for future work that we will pursue to uncover the properties of reciprocal networks so as to further understand the structural properties of directed graphs.

Acknowledgments. This research was supported in part by a Raytheon/NSF subcontract 9500012169/CNS-1346688, DTRA grants HDTRA1- 09-1-0050 and HDTRA1-14-1-0040, DoD ARO MURI Award W911NF-12-1-0385, and NSF grants CNS-1117536, CRI-1305237 and CNS-1411636. We thank the authors of [9] for the datasets and the workshop reviewers for helpful comments.

References

1. Gong, N.Z., Xu, W.: Reciprocal versus Parasocial Relationships in Online Social Networks. Social Network Analysis and Mining 4(1), 184–197 (2014)
2. Mislove, A., Marcon, M., Gummadi, K.P., Druschel, P., Bhattacharjee, B.: Measurement and Analysis of Online Social Networks. In: IMC 2007, pp. 29–42. ACM (2007)

[7] The authors in [9] stated similar conclusion

3. Kwak, H., Lee, C., Park, H., Moon, S.: What is Twitter, a Social network or a News Media? In: WWW 2010, pp. 591–600. ACM (2010)
4. Magno, G., Comarela, G., Saez-Trumper, D., Cha, M., Almeida, V.: New Kid on the Block: Exploring the Google+ Social Graph. In: IMC 2012, pp. 159–170. ACM (2012)
5. Garlaschelli, D., Loffredo, M.I.: Patterns of Link Reciprocity in Directed Networks. Phys. Rev. Lett. 93, 268701 (2004)
6. Jiang, B., Zhang, Z.-L., Towsley, D.: Reciprocity in Social Networks With Capacity Constraints. In: KDD 2015, pp. 457–466. ACM (2015)
7. Hai, P. H., Shin, H.: Effective Clustering of Dense and Concentrated Online Communities. In: Asia-Pacific Web Conference (APWEB) 2010, pp. 133–139. IEEE (2010)
8. Gong, N.Z., Xu, W., Huang, L., Mittal, P., Stefanov, E., Sekar, Song, D.: Evolution of the Social-Attribute Networks: Measurements, Modeling, and Implications Using Google+. In: IMC 2015, pp. 131–144. ACM (2015)
9. Gonzalez, R., Cuevas, R., Motamedi, R., Rejaie, R., Cuevas, A.: Google+ or Google-? Dissecting the Evolution of the New OSN in its First Year. In: WWW 2013, pp. 483–494. ACM (2013)
10. Wolfe, A.: Social network analysis: Methods and applications. American Ethnologist 24(1), 219–220 (1997)
11. Jamali, M., Haffari, G., Ester, M.: Modeling the Temporal Dynamics of Social Rating Networks Using Bidirectional Effects of Social Relations and Rating Patterns. In: WWW 2011, pp. 527-536. ACM (2011)
12. Li, Y., Zhang, Z.-L., Bao, J.: Mutual or Unrequited Love: Identifying Stable Clusters in Social Networks with Uni- and Bi-directional Links. In: Bonato, A., Janssen, J. (eds.) WAW 2012. LNCS, vol. 7323, pp. 113–125. Springer, Heidelberg (2012)
13. Schiberg, D., Schneider, F., Schiberg, H., Schmid, S., Uhlig, S., Feldmann, A.: Tracing the Birth of an OSN: Social Graph and Profile Analysis in Google+. In: WebSci 2012, pp. 265–274. ACM (2012)
14. Google+ Platform, http://www.google.com/intl/en/+/learnmore/
15. Google+, http://en.wikipedia.org/wiki/Google+
16. Kwak, H., Chun, H., Moon, S.: Fragile Online Relationship: A First Look at Unfollow Dynamics in Twitter. In: CHI 2011, pp. 1091–1100. ACM (2011)
17. Google+ New Feature, http://googledrive.blogspot.com/2012/10/share-your-stuff-from-google-drive-to.html
18. Clauset, A., Shalizi, C. R., and Newman, M. E. J.: Power-Law Distributions in Empirical Data. SIAM Review 51, 661–703 (2009)
19. Fitting Power Law distribution, http://tuvalu.santafe.edu/ aaronc/powerlaws/
20. Backstrom, L., Boldi, P., Rosa, M., Ugander, J., Vigna, S.: Four Degrees of Separation. In: WebSci 2012, p. 33–42. ACM (2012)
21. Leskovec, J., Kleinberg, J., Faloutsos, C.: Graphs Over Time: Densification Laws, Shrinking Diameters and Possible Explanations. In: KDD 2005, pp. 177–187. ACM (2005)
22. Kumar, R., Novak, J., Tomkins, A.: Structure and Evolution of Online Social Networks. In: KDD 2006, pp. 611–617. ACM (2006)
23. Kossinets, G., Watts, D.J.: Empirical Analysis of an Evolving Social Network. Science 311, 88–90 (2006)
24. Leskovec, J., Chakrabarti, D., Kleinberg, J., Faloutsos, C., Ghahramani, Z.: Kronecker graphs: An approach to modeling networks. The Journal of Machine Learning Research 11, 985–1042 (2010)
25. Google Strips Down Google Plus, http://blogs.wsj.com/digits/2015/11/17/google-strips-down-google-plus/