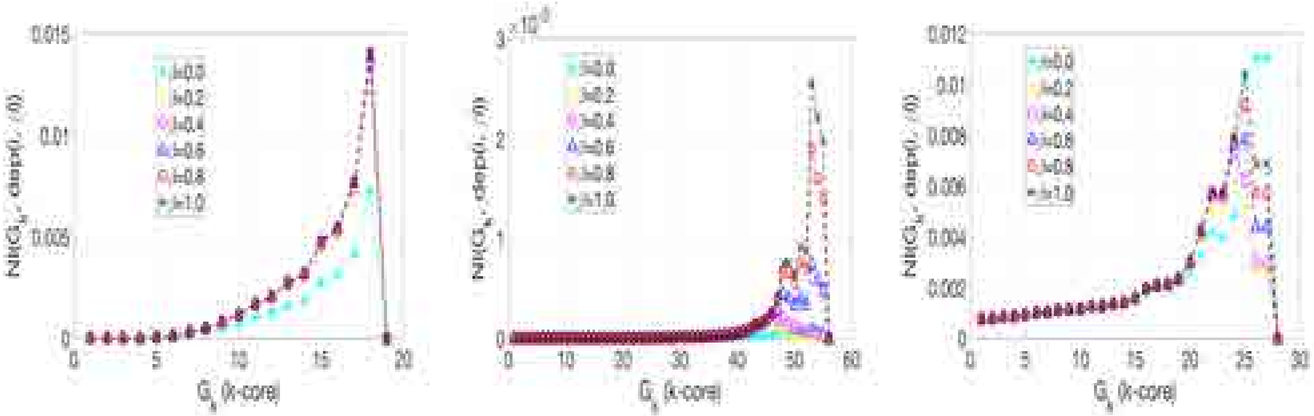
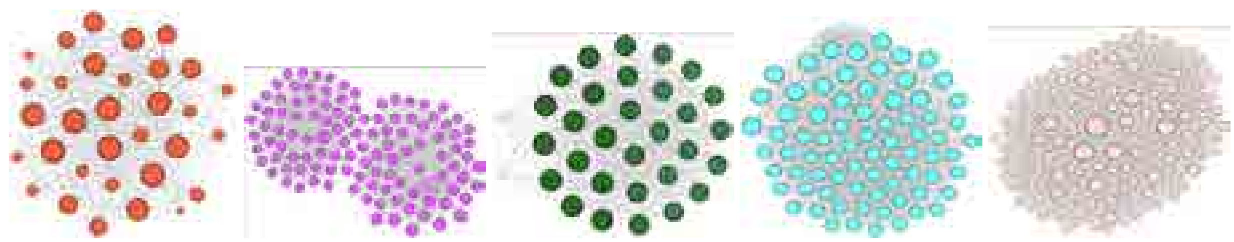


(a) Oregon-1 (b) ca-AstroPh (c) arenas-jazz

Figure 3: Variation of the nucleon-index per k-core index for several β parameters in the dependence computation



(a) Oregon-1 (b) ca-AstroPh (c) arenas-jazz (d) dnc-corecipient (e) email-Enron

Figure 4: Visualization of the core subgraphs: the size of a node is proportional to its degree

is a “dense core” of G, the product of these two terms should be large. The first term controls the rate of changes in size from G_k to G_{k+1}: intuitively, if G_k is the “nucleus” of G, going from G_{k-1} to G_k should not drastically change its size; but going from G_k to G_{k+1} amounts to breaking G_k apart, yielding a collection of small connected components. In other words, V_{k+1} would fall off quickly, as G_{k+1} is a small connected subgraph or an empty graph. Hence, G_k with the largest NI represents the *nucleus* of G (as produced by the decomposition process).

Considering the node dependence value as a centrality measure, we define $\theta(i)$ as follows:

$$\theta(i) := \frac{dep^{c(i)}(i, \beta)}{\sum_{j \in G} dep^{c(j)}(j, \beta)}. \quad (3)$$

Using $\theta(i)$ defined above and applying the nucleon-index to the k-shell decomposition procedure, we develop the following *stop rule* for core extraction.

Stopping rule for core extraction: For any graph G with a dense core structure, we should stop the k-shell decomposition method at the induced subgraph of the k_C-core with maximal nucleon-index. Thus, we seek a k_C-index that maximizes the nucleon-index (NI).

Figure 3 plots the nucleon-indices per k-core (C_k) for Oregon-1, ca-AstroPh and arenas-jazz networks. To select the optimal β parameter for eq. (1), we use the following criteria: let's assume that SK is the set of the k-indices corresponding to the maximum nucleon-indices, as β varies in the interval [0, 1] and k increases from 1 up to k_{max}. Then, we select any β associated with the k-index which appears most often in the set SK. For example, Table 2 shows the set SK for arenas-jazz. We select a β corresponding to the mode k_C-index value of 25 (i.e., $\beta = 0.1$; $\beta = 0.6$; $\beta = 1.0$).

Table 3 shows the (k_{max}, β, k_C) indices for our social network and Internet AS datasets and Fig. 4 provides a visualization of our extracted core subgraphs (G_C) for several example networks[3]. The smallest subgraph has 32 nodes and 362 edges (Oregon-1), whereas the largest one has 239 nodes and 28,441 edges (ca-HepPh). We will further investigate the structure of these core subgraphs (network nuclei) in the remaining sections.

4.3 Other Centralities and Nucleus

Nodes are more likely to be part of a network's core if they have high centrality score and if they are connected to other core nodes. Equation (2) can be used with a wide variety of $\theta(i)$ functions to transition between core and peripheral nodes. Thus, it allows one to use different ways to compute the nucleon-index (NI) and measure core quality. Here, we compute the nucleon-index using some of the most common centrality metrics: closeness centrality (c_c) [41,

[3] We omit the others plots here due to space constraint.

Table 3: maximum k-shell index (k_{max}); β parameter; k-index to stop the shells pruning process (k_C); number of nodes and edges in the core subgraph N(G_C) and E(G_C)

Network	k_{max}	β	k_C	$N(G_C)$	$E(G_C)$
arenas-jazz	29	0.6	25	32	466
dnc-corecipient	75	0.5	67	87	3,118
arenas-pgp	33	0.5	31	38	658
Oregon-1	20	0.5	18	32	362
ca-HepPh	238	0.5	99	239	28,441
ca-AstroPh	57	0.6	53	126	3,378
ca-CondMat	51	0.5	37	37	382
email-Enron	51	0.5	48	150	4,395
loc-brightkite	58	0.5	56	66	1,893
Facebook	64	0.5	61	285	9,616

Table 4: k-index to stop the shells pruning process (k_C) for several centralities: c_c - closeness centrality; b_c - betweenness centrality; e_c - eigenvector centrality; dep - dependence

Network	k_C			
	$\theta(i) = c_c$	$\theta(i) = b_c$	$\theta(i) = e_c$	$\theta(i) = dep$
arenas-jazz	26	25	26	25
dnc-corecipient	68	65	68	67
arenas-pgp	31	30	31	31
Oregon-1	18	18	18	18
ca-HepPh	99	99	99	99
ca-AstroPh	53	53	53	53
ca-CondMat	42	37	37	37
email-Enron	48	48	48	48
loc-brightkite	55	48	56	56
Facebook	60	60	60	61

42, 45], betweenness centrality (b_c) [14, 41, 45] and eigenvalue centrality (e_c) [12, 34, 41, 45] – we compare the obtained k_C-indices with the values computed in the previous section.

The closeness centrality measures how central a node is in terms of its distance (shortest path) from all other nodes [41], while the betweenness centrality for a node measures the number of shortest paths that pass through that node [41]. The eigenvalue centrality computes the centrality for a node based on the centrality of its neighbors. It is based on the notion that a node should be viewed as important if it is linked to other important nodes, where a node importance (or centrality score) corresponds to the largest eigenvector of the adjacency matrix [41]. Table 4 shows the k_C-indices for the different centrality measures and Fig. 5 plots the nucleon-indices versus k-core indices of several example networks[4]. In general, we observe that all the centralities give consistent k_C-indices or core structures for our datasets. In particular, we observe that our dependence metric, $dep(i, \beta)$, derives similar core structure when compared to the other metrics. From the consistency of the results given by the studied centrality metrics, we can infer that our social networks (see § 2) truly have a core structure.

[4]We omit the others plots here due to space constraint.

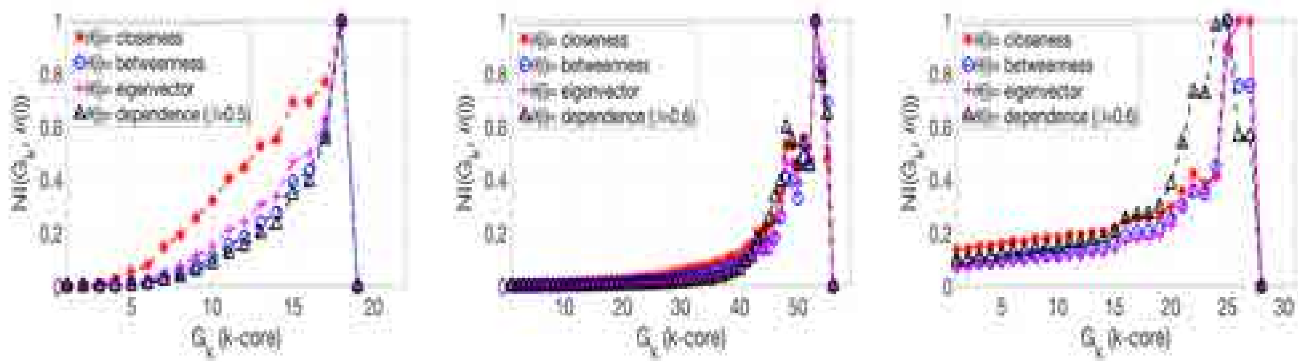
Table 5: Comparing classical k-shell decomposition (KS), Nucleon Index (NI) + k-shell decomposition (KS) and Rich-Club network core (G_C) in real-world networks : N - number of nodes; E - number of edges; D - diameter; P - path length; ρ - density

method	dataset	N	E	D	P	ρ
Classical KS	Oregon-1	20	164	2.0	1.14	0.86
	ca-AstroPh	17	136	1.0	1.00	1.00
	email-Enron	36	472	2.0	1.25	0.75
NI + KS	Oregon-1	32	363	2.0	1.27	0.73
	ca-AstroPh	126	3,378	3.0	1.87	0.43
	email-Enron	150	4,395	3.0	1.61	0.39
Rich-Club	Oregon-1	37	314	3.0	1.57	0.47
	ca-AstroPh	82	994	3.0	1.80	0.30
	email-Enron	106	1,660	4.0	1.77	0.30

All the centrality metrics discussed here are designed to measure notions of node importance in a network. Nevertheless, they have different computational complexity and require different network information. For example, the closeness and eigenvalue centralities need the full network information and have a high complexity of O(V^3). The betweenness centrality has a lower complexity of $O(VE)$ [14]. Our approach to calculate the $dep(v, \beta)$ score for node v is dependent on the k-shell decomposition method and degree computation which have a complexity of $O(V+E)$. Then, given that the degree and coreness of each node are known, our procedure has a complexity of $O(E)$. For a large sparse social network with $O(n)$ edges, this yields a linear time algorithm. Therefore, our methodology is highly scalable and can be applied to massive networks (hundreds million nodes and billion edges).

We compare our methodology to extract core subgraphs to the classical k-shell decomposition [15] and rich club [31, 51] methods. Table 5 provides statistics for the structure of the derived core subgraphs (G_C) for three of our networks (i.e., *Oregon-1, ca-AstroPh* and *email-Enron*) – we omit the others networks here due to space constraint. In general, for our dataset, we observe that the classical k-shell decomposition method (KS) is bias toward small and highly dense core subgraphs, G_C^{KS}, (i.e., a clique) which may not represent the "network core" (see § 3). In contrast, our modified k-shell decomposition method ($NI + KS$) generates larger core subgraphs than KS. In fact, our core subgraphs are supersets of the cores extracted using KS: $G_C^{NI+KS} \supset G_C^{KS}$. When compared to rich-club, we see that for some networks our modified k-shell decomposition method ($NI + KS$) generates core subgraphs of similar size (e.g., *Oregon-1*). However, our core subgraphs have more compact structure: small diameter, small path length and high density. For other networks, our methodology generates larger and denser core subgraphs than the rich-club method (e.g., *email-Enron*). This can be explained due to the fact that the rich-club is bias toward nodes with higher degree[5]. Differently, our definition of core is more general, and it allows low-degree nodes to belong to the core, as long as, they are important components in the structure of the network.

[5]Rich-club is a group of high-degree nodes in a network that preferentially connect to one another. This structure might be the core subgraph for power law networks



(a) Oregon-1 (b) ca-AstroPh (c) arenas-jazz

Figure 5: Variation of the nucleon-index (NI) per k-core index for several centrality metrics: the value of NI is normalized; the k-index to stop the shells pruning process (k_C) corresponds to the max(NI)

Table 6: Summary of path length (P) and diameter (D) characteristics: $\delta(u, G_C)$ - shortest path from node u to the core subgraph G_C

Network	P	D	$Avg(\delta(u, G_C))$
arenas-jazz	2.21	6	1.27
dnc-corecipient	2.27	8	1.63
arenas-pgp	7.65	24	4.27
Oregon-1	3.62	10	1.54
ca-HepPh	4.67	13	2.38
ca-AstroPh	4.17	14	2.24
ca-CondMat	5.35	14	3.25
email-Enron	4.03	13	1.74
loc-brightkite	4.92	18	3.41
Facebook	4.31	15	2.42

5 ANALYSIS OF THE NETWORK CORE STRUCTURE

Given the dense structures of our core subgraphs, illustrated in Figure 4, we now investigate the importance of this substructure for the network. To achieve this, we define and analyse the following metrics:

Core Path Length: To understand how much the network core contributes towards the small path lengths, we measure how many hops there are between any user to the core subgraph: $\delta(u, G_C) = min_{y \in G_C}\{d(u, y)\}$; $G_C \subset G$. Figure 6 presents the core path length and network path length distribuitions for *Oregon-1*, *ca-AstroPh* and *arenas-jazz*[6], whereas Table 6 shows the average values and the diameter for all the networks. From these results, we can see that most users are approximately 4 hops away from a random user and at most 2 hops away from the core (G_C), which implies that our core subgraphs are important structure for the connectivity of the nodes in the network.

[6] We obtain similar results for the other datasets. We omit the plots here due to space constraint.

Table 7: Ratio of the distance between nodes u and v to their respective distance to the core subgraph G_C: $R(u, v)$

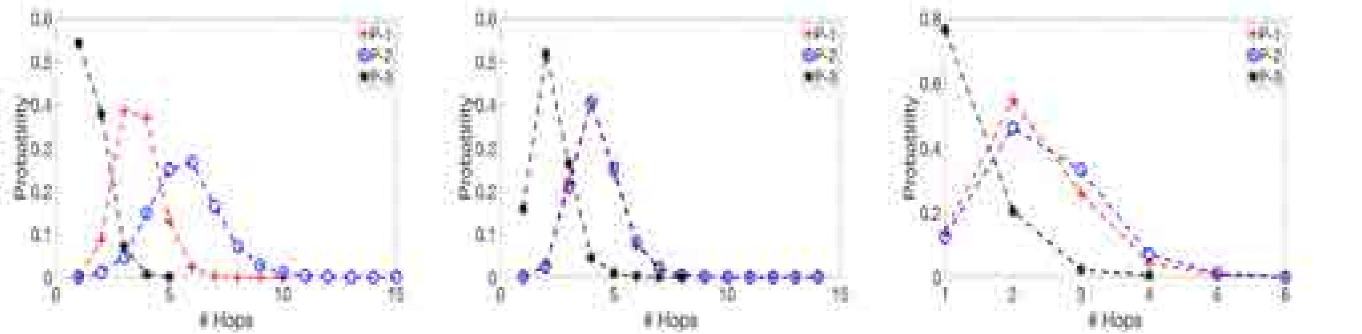
Network	k	Avg($R(u, v)$)
arenas-jazz	70	0.96
dnc-corecipient	700	0.90
arenas-pgp	8,000	0.89
Oregon-1	8,000	1.21
ca-HepPh	8,000	1.03
ca-AstroPh	8,000	0.96
ca-CondMat	20,000	0.84
email-Enron	20,000	1.21
loc-brightkite	20,000	0.73
Facebook	20,000	0.92

Core Centrality: We now investigate the importance of the core subgraph for communication and information diffusion in the network. To achieve this, we use the following procedure: first, we randomly sample k unique pairs of nodes (u, v). Then, we measure, $R(u, v)$, the ratio of the distance between nodes u and v to their respective distance to the core subgraph, as expressed in eq.(4), where $d(u, v)$ represents the shortest path between u and v, and $d(u, G_C)$ or $d(v, G_C)$ represents the shortest path between u or v to the core subgraph G_C.

Table 7 shows the average $R(u, v)$ for $k = 70$, $k = 700$, $k = 8,000$ and $k = 20,000$ respectively. We observe that the avg($R(u, v)$) is very close to the optimal value of 1.0, which implies that our core subgraph G_C contains the nodes with the highest *betweeness* in the network and they act as "bridges" for the connectivity between the other nodes in the network.

$$R(u, v) = \frac{d(u, v)}{d(u, G_C) + d(v, G_C)} \quad (4)$$

Core Removal: Lastly, we investigate the impact of removing the core subgraph G_C in the structure of the studied networks. We observe that all the neworks described in § 2 have a giant connected component (GCC) contaning more than 90% of all the nodes and more than 85% of all edges in the network. After the core removal,



(a) Oregon-1 (b) ca-AstroPh (c) arenas-jazz

Figure 6: Path length distribuitions: P-1: distance between nodes in the original network; P-2: distance between nodes in the original network, after core removal; P-3: nodes distance to the core subgraph G_C

we see that, for some networks (i.e., arenas-jazz, dnc-corecipient, Oregon-1 and email-Enron), at least 20% of the nodes break away from GCC, forming many isolated components of smaller sizes. Table 8 shows the number of these new connected components per network as well as the ratio of the size of the GCC after and before call removal in terms of the number of nodes and edges. From these results, we deduce that removing G_C significantly affects the connectivity and density for some of the networks.

Figure 6 shows the path length distribuition after we remove the core from our networks. We observe that the average path length increases after the core removal for most of the networks. For example, *ca-AstroPh*, *email-Enron* and *Oregon-1* have average path length of 4.17, 4.03 and 3.62 before core removal, and 4.25, 4.49 and 5.72 after core removal. This result provides further evidence that the core subgraph G_C is an important structure for reachability, communication and information diffusion in these networks. Next, we discuss the implications of our results.

Table 8: Basic stats of the giant (largest) connected components (GCC) after core removal: c_n - number of connected components; n_j and n_i- number of nodes in GCC before and after core removal; e_j and e_i - number of edges in GCC before and after core removal

Network	# c_n	n_i/n_j	e_i/e_j
arenas-jazz	2	0.833	0.612
dnc-corecipient	104	0.757	0.404
arenas-pgp	26	0.993	0.940
Oregon-1	3,183	0.688	0.503
ca-HepPh	73	0.967	0.645
ca-AstroPh	12	0.946	0.929
ca-CondMat	2	0.997	0.978
email-Enron	3,350	0.800	0.711
loc-brightkite	65	0.972	0.957
Facebook	66	0.994	0.930

6 DISCUSSION

Using examples from communication networks as well as collaboration, location-based, interaction, and online social networks, we have demonstrated that our method can effectively uncover and extract the nucleus of these networks. In this section, we discuss the limitations and implications of our method and results.

First, our proposed methodology to uncover the nucleus of networks can also be applied to weighted and directed networks by using a variation of the k-shell decomposition method: Garas et al. [24] presented a weighted k-shell decomposition method and Batagelj et al. [10] generalized the k-shell decomposition to directed networks. Our method can be applied with these generalized algorithms because our dependence and nucleon-index metrics are independent to the k-shell decomposition method. Once the k-shells are provided by decomposing the network into k-layers, the dependence and nucleon-index values can be computed.

Second, the "coreness" centrality or k-shell index has been argued to be a better measure than node degree for identifying influential spreaders in a network [23, 28]. However, our results show that using k-shell indices as a predictor of spreading influence of a node can be misleading. This is due to the fact that for a node to have a high k-shell index, it just needs to be a part of a very strong structure (e.g., a clique). This structure, however, may be isolated and lie at the edge or periphery of the network, instead of its core (see § 3). Our analysis shows that the dependency value of a node, $dep^k(i)$, provides important information about the structure function of each node in the graph. Thus, we believe that by using a node dependency value along with its k-shell index (dep^k, k), we can better predict the spreading influence of a node than simply using its k-shell index. We will investigate this in the future.

Third, unveiling the core structure of social networks may have implications in the design of algorithms for information flow, and in development of techniques for analysing the vulnerability or robustness of networks. In addition, analysis of the core structure of social networks can help us uncover and understand possible organizing principles shaping the observed network topological structure and network formation.

7 RELATED WORK

In contrast to the wealth of attention given to community structure analysis in the literature, there are comparatively few methods for extracting and analyzing the core structure of a network. Some studies simply define the network "core" as the maximal clique composed of the highest degree nodes in a network [44], while other studies focus instead on some notion of connectivity information (e.g. betweeness, closeness, etc.) to find the core and periphery of a network [16, 17, 27, 33, 43].

One of the most popular quantitative methods to investigate core-periphery structure was proposed by Borgatti and Everett in 1999 [13]. Based on this study, several methods for identifying the core-periphery of a network have been proposed [16, 17, 27]. These algorithms attempt to determine which nodes are part of a densely-connected core and which are part of a sparsely connected periphery by solving some complex optimization problem. Consequently, most of these methods are computationally expensive and do not scalable to large networks.

The authors in [47] used the notion of α-β community to extract the "core" of a graph. An α-β community is a connected subgraph C with each vertex in C connected to at least β vertices of C and each vertex outside of C connected to at most α vertices of C ($\alpha < \beta$). They extract the network core structure by taking the intersection of α-β communities of different size k. A core thus corresponds to one or multiple dense regions of the graph. As a result, the proposed heuristics in [47] may return multiple dense regions ("cores") for a given network. In addition, this algorithm does not guarantee to terminate within a reasonable amount of running time.

8 CONCLUSION

In this paper, we have advanced and developed an effective procedure to extract the *core* structure of social networks. First, we introduce a new metric – the node "dependence value" – that measures the location importance of a node in a network. Second, we define a new measure called *nucleon-index* that captures the extend to which a subgraph is a densely intra-connected and topological central core. Then, using these metrics, we proposed a modified version of the k-shell decomposition method by identifying the k_C-index where we should stop pruning the network in order to preserve its core structure. For our social network datasets, we found that they contain very dense core subgraphs G_C. The smallest core has 32 nodes and 362 edges (Oregon-1), whereas the largest one has 239 nodes and 28,441 edges (ca-HepPh). Finally, given a dense core subgraph G_C, we investigate the importance of this substructure for the network by analysing the following metrics: i) the distance between a node v to the core subgraph G_C; ii) the ratio of the distance between nodes u and v to their respective distance to G_C and iii) lastly, the impact of removing G_C in the structure of the network G ($G_C \subset G$).

As part of ongoing and future work, we will provide a more in-depth analysis of the dense core subgraph G_C of social networks. We also plan to apply our method to a massive Google+ dataset [19, 20, 26] (with more than 170 million nodes and $\approx$ 3 billion edges), a massive Twitter dataset [21] (with more than 500 million nodes and $\approx$ 23 billion edges) and other social networks.

ACKNOWLEDGMENTS

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9 APPENDIX

Beta Parameter Selection: We now establish that the contribution of the h-step removed neighbors of node i is attenuated by β^{h-1}:

Given that $dep^0(i) = 0$ and $dep^1(i) = \delta^1(i)$, we can write an expression for $dep^2(i)$ as following:

$$\begin{aligned} dep^2(i) &= dep^1(i) + \delta^2(i) + \beta \times \Sigma_{j \in N^2(i)} dep^1(j) \\ &= \delta^1(i) + \delta^2(i) + \beta \times \Sigma_{j \in N^2(i)} \delta^1(j) \end{aligned} \quad (5)$$

Let us assume that node i has $c(i) = 4$, then $dep^4(i)$ is computed as following:

$$dep^4(i) = dep^3(i) + \delta^4(i) + \beta \Sigma_{j \in N^4(i)} [dep^3(j)] \quad (6)$$

Expanding eq. (6) yields:

$$\begin{aligned} dep^4(i) = dep^3(i) + \delta^4(i) + \beta \Sigma_{j \in N^4(i)} [dep^2(j) + \delta^3(j) \\ + \beta \Sigma_{j' \in N^3(j)} dep^2(j')] \end{aligned}$$

Substituting eq. (5) yields:

$$\begin{aligned} dep^4(i) := dep^3(i) + \delta^4(i) + \beta \Sigma_j [M^3(j) + \beta \delta^2(j) \rho^1(j'^*) \\ + \beta \Sigma_{j'} [M^2(j') + \beta \delta^2(j') \rho^1(j'')]] \end{aligned}$$

where $M^k(i) = \Sigma_k \delta^k(i)$ and $\delta^k(i) = \rho^k(i), \forall i \in V$.

Further simplify $dep^4(i)$ yields:

$$\begin{aligned} dep^4(i) := dep^3(i) + \delta^4(i) + \Sigma_j [\beta M^3(j) + \beta^2 \delta^2(j) \rho^1(j'^*) \\ + \Sigma_{j'} [\beta^2 M^2(j') + \beta^3 \delta^2(j') \rho^1(j'')]] \end{aligned}$$

We can rewrite the above expressions as:

$$dep^4(i) := dep^3(i) + \beta^0 A + \Sigma_j [\beta B + \beta^2 C + \Sigma_{j'} [\beta^2 D + \beta^3 E]] \quad (7)$$

where:

- $A = \delta^4(i)$: 1-step neighbors of i removed at $k = 4$
- $B = M^3(j)$: 2-step neighbors of i removed at $k = 1, 2, 3$
- $C = \delta^2(j) \rho^1(j'^*)$: 3-step neighbors of i removed at $k = 1$
- $D = M^2(j')$: 3-step neighbors of i removed at $k = 1, 2$
- $E = \delta^2(j') \rho^1(j'')$: 4-step neighbors of i removed at $k = 1$

By generalizing eq. (7) ($k = 5, ..., n$), we observe that at every k-index, the number of h-step removed neighbors of i is multiplied by β^{h-1}. This concludes our proof. As stated before, the parameter β quantifies the contribution of node j to the total dependence value of node i. Thus, by varying β, we are impacting the contribution of any node j to the total dependence value of node i by the same proportion.

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