The Effect of Different Couplings on Mitigating Failure Cascades in Interdependent Networks

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***Abstract*—Many real-world (cyber-)physical infrastructure systems are multi-layered, consisting of multiple *inter-dependent* networks/layers. Due to this interdependency, the failure cascade can be catastrophic in a inter-dependent, multi-layered system, and even lead to the break-down of the entire system. The 2003 blackout of the Italian power grid is reportedly the result of a cascading failure due to the inter-dependency of the power grid and the communication network that it relies on. In this paper, we propose a theoretical framework for studying cascading failures in an inter-dependent, multi-layer system, where we consider the effects of cascading failures both *within* and *across* different layers. The goal of the study is to investigate how different *couplings* (i.e., inter-dependencies) between network elements across layers affect the cascading failure dynamics. Through experiments using the proposed framework, we show that under the one-to-one coupling, how nodes from two inter-dependent networks are coupled together play a crucial role in the final size of the resulting failure cascades: coupling corresponding nodes from two networks with equal importance (i.e., "high-to-high" coupling) result in smaller failure cascades than other forms of inter-dependence coupling such as "random" or "low-to-high" coupling. Our results shed lights on potential strategies for mitigating cascading failures in inter-dependent networks.**

I. Introduction

We now live in an increasingly *connected* world which hinges critically on many *inter-dependent* cyber-physical infrastructure systems. These systems include (smart) power grids, intelligent transportation systems, communication networks and the global Internet. These infrastructures rely on computer and control systems as well as communication networks to sense, collect, estimate the system state, environment and other information, invoke and execute appropriate computations and control strategies to adjust and adapt to changes in the system state and to actuate the physical system components to respond to such changes. The cyber system components also serve as a crucial interface between the physical system components and human operators (as well as end users who are ultimate producers/consumers of much of the information, services or goods that the cyber-physical infrastructures provide).

The inter-dependence of critical cyber-physical infrastructure systems is perhaps best exemplified by the relations between power grids and communication networks where power grids rely on communication networks to deliver the state information of the power system to the control system and relay control back to the power system, while the communication networks depend on the same power grids for the electrical supply. Due to such interdependence, element faults in one network, e.g., crashes of a few switches in the communication network that is used to relay information and control to a smart grid, can induce failures in the other, i.e., the power grid, which would in turn lead to additional failures in the communication network, thereby triggering a cascade of failures in these two *inter-dependent* networks. It has been reported that a number of electrical blackouts, such as the one in Italy on 28 September 2003 [1], have in fact been caused by such inter-dependency induced cascaded failures.

We note that the phenomenon of cascading failures can occur in a *single* network. For example, cascading failures occur frequently in a power grid due to the physical nature of the system as failures of transmission lines or power generators can trigger additional node or line failures due to load imbalance or thermal effect. In a communication network, network element (router or link) failures will trigger network control elements to exchange route control messages and re-compute routes to re-route traffic around failed links/nodes; cascading failures may be triggered due to excessive route re-computation overloads at surviving network elements, which lead to further failures. In a *multi-layered* system of *inter-dependent networks*, failures of network elements in one constituent network (also simply referred to as one layer of the multi-layered system) may not only trigger cascades with the same layer, but also trigger failures of network elements in other constituent networks (layers) of the system. Inter-dependencies across the constituent networks of a multi-layered system can induce cascading failures with very different characteristics and dynamics than those occurring within only one layer, often causing wider and more severe damages to the overall system. To assess and enhance the resiliency of a multi-layered systems of inter-dependent networks, it is therefore imperative to understand how inter-dependencies affect cascading failures within and across constituent networks in a multi-layer system.

In this paper we propose a theoretical framework for studying cascading failures in an inter-dependent, multi-layer system, where we consider the effects of cascading failures both *within* and *across* different layers. The goal of the study is to investigate how different *couplings* (i.e., inter-dependencies) between network elements across layers affect the cascading failure dynamics. For simplicity of exposition, we consider a two-layer system with two constituent networks of equal size, and adopt a simple *one-to-one* coupling map across the two layers. Cascading failures within each layers are modelled using the standard *linear threshold* model[1]. We examine how

[1] We remark that our theoretical framework can be applied to (or generalized to) multi-layer systems with more than two networks with more complex coupling functions and cascading failure models.

coupling of nodes of different "importance" or "criticality" (as measured by various metrics e.g., by node degree) from the two constituent networks affect the cascading failure dynamics under varying initial failure sizes and cascading thresholds within each layer. We show that under the *one-to-one* coupling map, how nodes from two inter-dependent networks are coupled together play a crucial role in the final size of the resulting failure cascades: coupling corresponding nodes from two networks with equal importance (i.e., "high-to-high" coupling) result in smaller failure cascades than other forms of inter-dependence coupling such as "random" or "high-low" coupling. In particular, given a two-layered system with two identical networks, "high-to-high" coupling produces a *mirror* effect in that the coupling exactly mirrors the cascade within each layer and does not produce additional failures than when the two networks are independent.

II. Related Work

Due to its increasing importance, resilience of inter-dependent networks has attracted a flurry of interest from a broad and diverse array of research communities. Using a percolation theory-based framework with random graph models, Buldyrev et al [2] demonstrate that interdependent networks can behave very differently from each of their constituents. In their work – and those of many others, the "robustness" of interdependent networks is quantified in terms of asymptotic statistical properties such as the existence of *giant connected components* under random failures. It is well known from the theory of complex networks that (an ensemble of random) power-law networks are more resilient to random node failures, as there is a phase transition in the fraction of random node failures, below which the giant connected component exists with high probability. In [2] Buldyrev et al show that when nodes from two "robust" power-law networks are *randomly coupled* together *one-to-one*, they become more vulnerable to random failures in the sense that no giant connected component exists with high probability under any fraction of random node failures. In a follow-up work, Parshani et al. [3] show that decreasing the interdependency of the layers, by decoupling some nodes (as are called autonomous) which do not require any resource from the other layers, the failure cascade can be mitigated. In this work, the nodes were picked *randomly* to become autonomous nodes. Schneider et al. [4] suggest a *centrality* based method for picking the autonomous nodes and show how effectively this method reduces the number of required autonomous nodes by a factor of five compared with the random method. In another work, Brummit et al. [5] pursue the Bak-Tang-Wiesenfeld sandpile model [6] to study failure cascades in inter-dependent networks. They show that adding a few interconnections between the layers of the network is beneficial, but it becomes destructive if the number of interconnections are too many. They find the optimal degree of interdependency in which the failure cascade is minimized.

As in the case of robustness of single networks, the aforementioned characterizations of inter-dependent networks based on random graph models/percolation theory provide useful insight into the *general statistical properties of interdependences* over ensembles of *random* graphs/networks. In practice, however, real networks are *deterministic* and *finite*. In particular, *engineered* infrastructure networks such as power-grids and communication networks, are designed to perform certain specific functions, many of which arguably do not follow the "power-law" degree distribution. Furthermore, although the degree of interdependency is important in controlling failure cascades in interdependent networks, it is not always the case to be able to determine the number of autonomous nodes and in some applications this number is given (the resources are limited). In those cases, designing the way that non-autonomous nodes from different layers are coupled together is another effective solution to control and mitigate failure cascades. Rosato et al. [1] conduct a focused study of the inter-dependency between the Italian power grid and Italian communication network, where they demonstrate that line failures in the Italian power grid network can severely affect the Italian communication network even in the case of moderate interconnection of these two networks. In their study, the authors assume that the nodes in the Italian communication network draw power supply from the *geographically* close nodes in the Italian power grid network. In [7] Ranjan and Zhang proposal a graph-theoretical *finite network* model for representing inter-dependent networks and extend the *structural/topological centrality* measure [8] to develop a robustness metric of inter-dependent networks. Using this robustness metric, they show that both the number of coupled nodes from two inter-dependent networks and how they are coupled together can play a critical role in determining the overall robustness of inter-dependent networks. In [9] Nguyen et al study the *Interdependent Power Network Disruptor* (IPND) optimization problem to identify critical nodes in an interdependent power network whose removals maximally destroy its functions. Our work differs from these existing studies in that we not only consider the effects of cascading failures both *within* and *across* different layers, but also investigate how different ways of interdependency ("coupling") affect failure cascades in inter-dependent network. We evaluate the results on both real and synthetic networks.

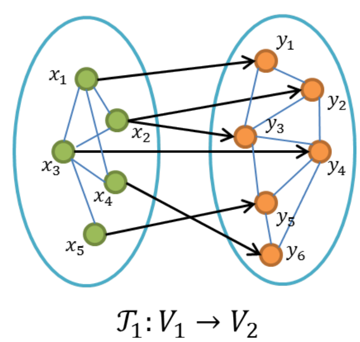

Fig. 1: Bijective inter-connection of layer 1 to layer 2

III. Failure Cascade Model

Consider a network $G(V,E)$, where V is the set of nodes and E is the set of edges. A failure cascade is initiated from a subset of nodes and yields a (larger) set of failed nodes. The failure cascade can be modeled as follows:

$$\mathcal{F} : \mathbb{P}(V) \rightarrow \mathbb{P}(V), \quad (1)$$

where $\mathbb{P}(V)$ is the power set of nodes and $\mathcal{F}$ is the failure function in this network. $\mathcal{F}$ depends on the connectivity of the nodes (network topology) and how the failure cascades through the network. In most real networks, when a node losses a *majority* of its connections to other nodes, the node practically becomes nonfunctional, thus "fails". The linear threshold (LT) model [10] well captures this phenomena in which node i is considered to have failed when the portion of its neighboring nodes $N(i)$ which have failed is larger than some threshold θ:

$$\Sigma_{j \in N(i)} w_{ji} \delta(j) \geq \theta, \quad (2)$$

where w_{ij}'s are *importance* weights assigned to the neighboring nodes. In the case of uniform weighting, $w_{ij} = 1$. Considering the LT model as the cascading function, the failure in one network starts from a set of failed nodes and cascades through the network in accordance with eq. (2). Note that the failure is considered to be *progressive*, namely when a node fails it does not recover throughout the process [10]. (For the non-progressive LT model, please refer to [11].) In a progressive cascading model, $\mathcal{F}$ is determined deterministically for a fixed θ and a given network G.

Real systems are not always as simple as a single layer network described above. They possess more complex structures, comprised of more than one network (or layer), where nodes in one layer require resources (i.e., power) from nodes in other layers, and in turn supply resources (e.g., control) to nodes in other layers. Such networks, in which the layers are inter-connected to each other, are referred to as *interdependent* networks. In an interdependent network, a node failure in one layer causes its *dependent* nodes in other layers (i.e., those relying on the resources supplied by the failed node to function) also to fail. For example, in fig. (1) if node x_2 fails, its dependent nodes in the other layer, i.e. y_2 and y_3 fail as well. Thus, in interdependent networks an initial failure in one layer may not only cause a failure cascade within the same layer, but also can triggers failure cascades in other layers. The failure cascadse in other layers in turn trigger further failures in the original layer, creating a "vicious cycle" which may lead to the break-down of the entire system. While interdependency in such networks is inevitable, it is possible to carefully "design" the inter-connections between the layers so as to mitigate the effects of failure cascades.

For this purpose, in this paper we propose a theoretical framework to model and study failure cascades in interdependent networks. Unlike a single layer network, we argue that in modeling inter-dependent networks, it is important to distinguish the functionality of "inter-connecting links" (interdependencies) between nodes across layers from the regular links between nodes within a single layer, *as the failure cascading processes within a single failure and across layers are general very different.* For example, failure of a node in general does not automatically leads to the failure of its neighboring nodes *within the same layer* (unless a large portion of neighboring nodes fail under the LT model discussed earlier). On the other hand, failure of a node (i.e., a power supply node) will cause its dependent nodes (e.g., communication or control nodes) in other layers to become non-functional, thus "fail" (with high probability), unless certain protection mechanisms (e.g., backup power) are provisioned. Even in the latter case, such protection mechanisms are often temporal and simply delay the potential failure if the failed nodes are not restored and recovered in time. We present the following general failure cascade model for an interdependent network with two layers $G_1(V_1,E_1)$ and $G_2(V_2,E_2)$, where $\mathcal{F}$ represents the function modeling the failure cascade *within* a layer and $\mathcal{T}$ the function modeling the failure cascade *across* the layers:

$$\begin{aligned} \mathcal{F}_1 &: \mathbb{P}(V_1) \rightarrow \mathbb{P}(V_1), \\ \mathcal{F}_2 &: \mathbb{P}(V_2) \rightarrow \mathbb{P}(V_2), \\ \mathcal{T}_1 &: V_1 \rightarrow \mathbb{P}(V_2), \\ \mathcal{T}_2 &: V_2 \rightarrow \mathbb{P}(V_1). \end{aligned} \quad (3)$$

Functions $\mathcal{F}_1$ and $\mathcal{F}_2$ are not necessarily injective or surjective. Fig. (1) illustrates a bijective function $\mathcal{T}_1$ from layer 1 to layer 2 ($\mathcal{T}_2$ is not shown).

In this paper, we show how a proper choice of the functions which model failure cascades *across* the layers can have a significant impact on (triggering/mitigating) the overal failure cascades across the layers. For the ease of exposition, we consider only a bidirectional $\mathcal{T}$ instead of two separate unidirectional $\mathcal{T}_1$ and $\mathcal{T}_2$. In other words, we assume that every node in each layer is served by a unique node (in the other layer) on which it relies for its resources but also for which it supplies the required resources. To have the bijective property in both directions, $\mathcal{T}$ is a "one-to-one" node mapping (or *coupling*) between the two layers, i.e. $\mathcal{T} : V_2 \leftrightarrow V_1$.

IV. Experiments and Results

In this section, we investigate the effect of failure cascade modeling functions, i.e. $\mathcal{T}$ and $\mathcal{F}$ in eq. (3), on failure cascades across the layers of an interdependent network. Using the LT model as the cascading function within a layer, $\mathcal{F}$ is a function of the threshold θ. For the interdependency ("coupling") function $\mathcal{T}$, we study three representative ways of coupling: 1)"high-to-high" degree coupling, in which the nodes in each layer are sorted based on their degree and are coupled to their corresponding (the same rank) nodes in other layers, 2)"high-to-low" degree coupling with pairing the node in a reverse ordering of their degree, and 3)"random" coupling.

We conduct a number of experiments for a wide range of the LT threshold values ($\theta \in [0.1, 0.9]$) and initial failure sizes ($s_{init} \in [1,n]$, n is the number of nodes in each layer). For a fixed size s_{init} of an initial failure, we pick a random s_{init} number of nodes as the initiators of the failure. However,

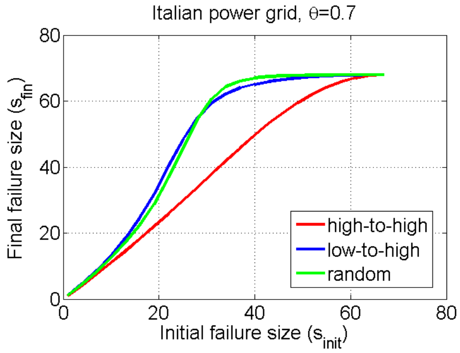

Fig. 2: Failure cascade in Italian power grid interdependent network for a fixed threshold and three different coupling.

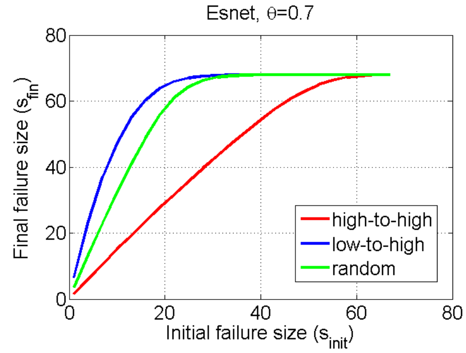

Fig. 3: Failure cascade in Esnet interdependent network for a fixed threshold and three different coupling.

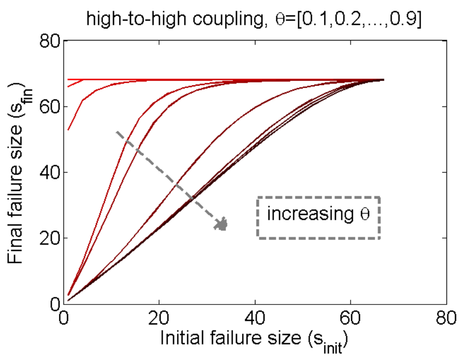

Fig. 4: Failure cascade in Italian power grid interdependent network for a range of thresholds and high-to-high coupling.

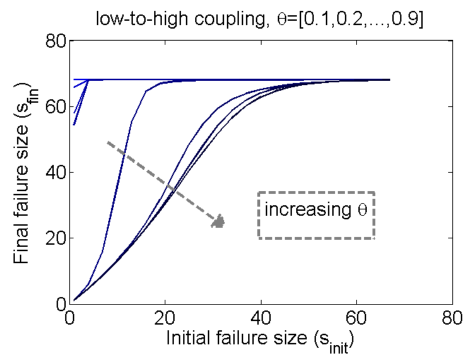

Fig. 5: Failure cascade in Italian power grid interdependent network for a range of thresholds and low-to-high coupling.

nodes possess different topological importance (centrality), the failure of which can lead to varying sizes of failure cascades (within each layer). Therefor, for each s_{init} we simulate the failure cascades for 10,000 random instance initiators and report the average failure size. Fig. (2) shows the results of failure cascades in the Italian power grid network [1] ($n = 68$), when it is coupled with a copy of its own. The experiments are conducted for a fixed threshold of $\theta = 0.7$ in both layers and the results are reported in terms of number of nodes failed in one layer at the end of the cascade process (due to one-to-one coupling, the number of failed nodes are equal in two layers at the end of the cascade). We also perform the exact same set of experiments on the Esnet network, the US DoE energy science network with $n = 68$ number of nodes. The results are reported in fig. (3). From figs. (2) and (3), we see that "high-to-high" coupling show enormously better performance in mitigating the failure cascade than the "low-to-high" and "random" coupling; while "high-to-high" curve is very close to the line $s_{fin} = s_{init}$, two other couplings result in 150% increase in the final failure size over the initial size in some instances for the Italian power grid case (even worse for the Esnet case). The line $s_{fin} = s_{init}$ (not shown in the figures) represents the case where the failure does not cascade and the final failure size is equal to the initial failure size. We also present further failure results for a range of thresholds $\theta \in [0.1, 0.9]$ for the Italian power grid interdependent network in figs. (4) and (5) for the cases of "high-to-high" and "low-to-high" couplings respectively. Comparing these two figures, it can be inferred that "high-to-high" coupling outperforms "low-to-high" coupling for every θ. Furthermore, increasing θ results in smaller failure cascade sizes, while increasing the initial failure size leads to larger failure cascades. (Due to space limitation, we omit reporting the corresponding results for the case of Esnet, which are very similar.)

Figs. (6) and (7) reflect the same experiment results explained above, but have been depicted in different way. To avoid making the figures crowded, we have presented the curves for every three other value of s_{init} from 16 to 34. It can be seen that for every s_{init}, s_{fin} follows a sigmoid-like function in terms of $1-\theta$: there exists one transition point before which the rate of growth is increasing (convex function) and after which the rate of growth is decreasing (concave function). The sigmoid behavior of failure cascades implies that decreasing the threshold up to some transiting point accelerates the failure cascade, but passing that point the rate of cascade slows down. The figures suggest that the transition point is independent of s_{init}; it happens around $\theta \simeq 0.55$ for "high-to-high" coupling and around $\theta \simeq 0.65$ for "low-to-high" coupling. The following general function captures the sigmoid behavior of the final

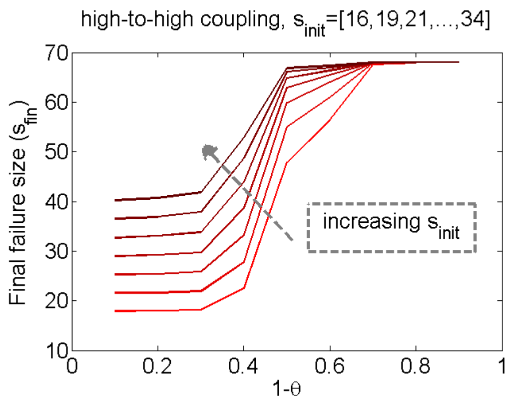

Fig. 6: Failure cascade in Italian power grid interdependent network for a range of initial failure size and high-to-high coupling

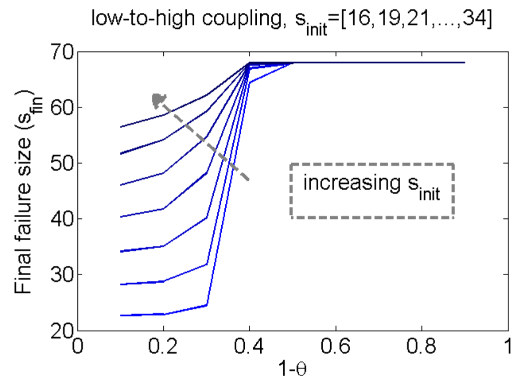

Fig. 7: Failure cascade in Italian power grid interdependent network for a range of initial failure size and low-to-high coupling.

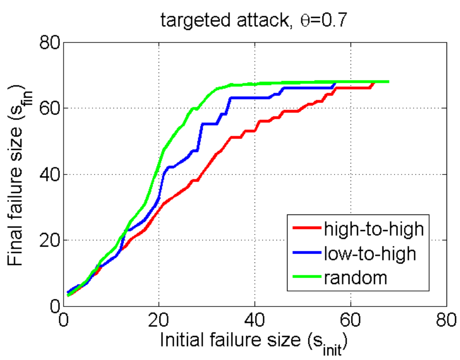

Fig. 8: Failure cascade in Italian power grid interdependent network for a targeted attack and fixed threshold.

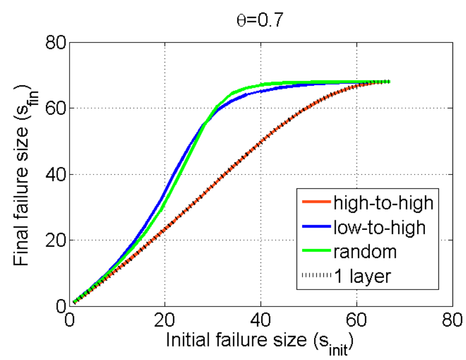

Fig. 9: Mirroring effect of failure cascade in Italian power grid interdependent network.

failure size:

$$s_{fin} = \frac{n - g_1(s_{init})}{1 + \exp(-g_2(s_{init})(g_3(\theta)))} + g_1(s_{init}), \quad (4)$$

where g_1, g_2, and g_3 are linear functions. For example, for "high-to-high" coupling in the Italian power grid network, these g functions are best fitted with the following linear functions: $g_1(x) = 1.25x - 2$, $g_2(x) = \frac{1}{9}x + \frac{2}{9}$, and $g_3(x) = -10x + 5$. The closed form formulation presented in eq. (4) can be useful in predicting the failure size for the large real networks where the simulation is costly or even infeasible in some cases.

Up to now, all the experiments presented in this section have been designed for initial *random* failure. Fig. (8) shows the failure result when the initial failure is *targeted*: namely, the failures of more important and central nodes are the results of a targeted attack. In these experiments, the nodes with higher degree are considered to be the initial set of failed nodes. Studies [12] show that the targeted attacks in real networks, where the degree distribution follows a power law distribution, are more harmful than random attacks. Our experiment results show that the "high-to-high" coupling in interdependent network outperforms the other two couplings in targeted attacks as well and assures higher resilience to failure cascades. The failure results obtained for "random" coupling are the average of 10,000 experiments of randomly coupling nodes in the two layers.

As discussed in the previous section, without the interdependency the failure cascade may be minimum in each layer, which is the result of some initiated failure in that layer (i.e., only $\mathcal{F}_1$). Failures in interdependent networks, on the other hand, can cause a "vicious" cycle: when a failure occurs in one layer, besides cascading through the same layer ($\mathcal{F}_1$ in eq. (3)), it triggers failures in other layers ($\mathcal{T}_1$); These failures in turn cause further failures in the original layer ($\mathcal{F}_2$ and then $\mathcal{T}_2$) – this cycle continues. To investigate the effects of different couplings in triggering/mitigating failure cascades, they should be compared against the failure cascade in a single layer network. In fig. (9) we compare the failure cascade for the three coupling cases against the failure cascade in one-layer network. This experiment is the same as the experiment in fig. (2) but adding the result of the *least possible* failure cascade as well, i.e. failure cascade in one-layer Italian power grid network. It can be seen that, interestingly, the "high-to-high" coupling is in fact equal to having no interdependency at all. This happens due to the *mirroring effect* in which the coupling exactly mirrors the cascade in the two layers and does not lead to further failure than the one is already happening in each of

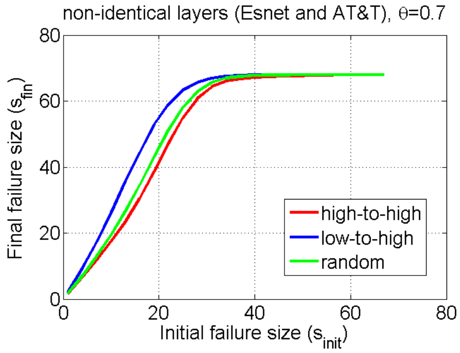

Fig. 10: Failure cascade in an interdependent network with Italian power grid network as one layer and Esnet as the other.

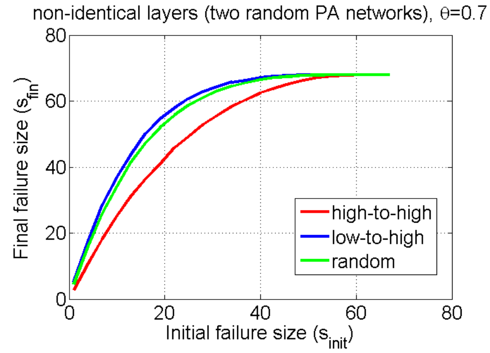

Fig. 11: Failure cascade in an interdependent network with two layers generated by preferential attachment,

the layers. Thus, leveraging the mirroring effect we are able to design the interdependency functions to minimize the failure cascade in interdependent networks. In the case of identical layers (i.e. layers with the same topology), the best coupling is to pair congruent (equivalent) nodes of the two layers which is the same thing done in "high-to-high" coupling in our experiment fig. (9). However, when the layers are not identical, it is more complicated to find the optimum solution. In this case, we should find the best alignment of the layers to benefit from the mirror effect the most possible. We have conducted two experiments on two interdependent networks with non-identical layers: 1) Italian power grid network coupled with Esnet network (fig. (10)), and 2) two networks generated by preferential attachment model [13] with the same size of n=68 nodes (fig. (11)). The figures indicate that the "high-to-high" coupling outperforms the other two couplings, suggesting that "high-to-high" coupling is more successful in *imitating the mirror effect*, i.e., coupling the congruent nodes of the layers in these experiments.

V. Conclusion

In this paper, we have developed a theoretical framework for studying cascading failures in an inter-dependent, multi-layer system, where we consider the effects of cascading failures both *within* and *across* different layers. The goal of the study is to investigate how different *couplings* (i.e., inter-dependencies) between network elements across layers affect the cascading failure dynamics. Through experiments using the proposed framework, we show that under the one-to-one coupling map, how nodes from two inter-dependent networks are coupled together play a crucial role in the final size of the resulting failure cascades: coupling corresponding nodes from two networks with equal importance (i.e., "high-to-high" coupling) result in smaller failure cascades than other forms of inter-dependence coupling such as "random" or "high-low" coupling. In particular, given a two-layered system with two identical networks, "high-to-high" coupling produces a *mirror* effect in that the coupling exactly mirrors the cascade within each layer and does not produce additional failures than when the two networks are independent. Our results shed lights on potential strategies for mitigating cascading failures in inter-dependent networks, and also pose interesting and important new research questions.

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