ECE 5210

Theory of Linear Systems

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1 Course description

Linear systems are dynamical systems whose evolution is defined through state space equations of the form

$$\begin{cases} \dot{x}(t) = A(t)x(t) + B(t)u(t) \\ y(t) = C(t)x(t) + D(t)u(t) \end{cases} \quad \text{or} \quad \begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = C(k)x(k) + D(k)u(k) \end{cases} \tag{1}$$

depending on whether the system is described in continuous (left) or discrete time (right). Linear systems of this form are useful in many application areas: They frequently arise as models of mechanical or electrical systems and also arise when one linearizes a non-linear system around a desired trajectory.

This course has two main objectives. The first (and more obvious) is for students to learn something about linear systems. For example we will start by proving that the solution to the systems in Eq. (1) exists and is unique. This is a fundamental prerequisite, since our objective is to use Eq. (1) to model a physical system (an airplane, a car, a circuit etc.). If the differential equation used to model the system does not have solutions or has many, this is an indication that the model we developed is inadequate, since it cannot tell us what the physical system is "going to do" as time progresses. After we established that the model in Eq. (1) is a good abstraction, we will learn how to use such a model to understand properties of interest for the physical system it represents. For example, we will study if and how the control input $u(t)$ can be used to influence the system dynamics to a desired objective (e.g., how can we use the steering wheel and pedals to park a car) and whether the observed output $y(t)$ is enough to tell us something about the internal state of our system (e.g., if observing the number of hospitalizations is enough to tell us how many infections there are during an epidemic). We will then turn to questions about the future behavior of the physical system and will study how to use the model in Eq. (1) to predict what will happen asymptotically (as time tends to infinity) by introducing the key concept of stability. Time permitting, we will finally discuss more advanced topics such as the Kalman filter, linear quadratic regulator and H_∞ control, which overall should be seen as control tools to influence the physical system to a desired behavior.

The second and less obvious objective of the course is for students to experience something about doing automatic control research, in particular developing mathematical proofs and formal logical arguments. Linear systems theory is ideally suited for this task and almost all the derivations given in the class will be carried out in complete detail, down to the level of basic algebra.

A tentative list of topics covered in the course include

- State space modeling of dynamical systems in continuous and discrete time;
- Existence and uniqueness of the solution to time-varying linear systems;
- Time invariant linear systems and the matrix exponential (Cayley-Hamilton theorem, Laplace and Z transform);
- Controllability, reachability, observability;
- Lyapunov stability of linear systems; Stability in continuous and discrete time;
- Feedback controllers and observers;
- Realization theory;
- Time permitting topics from: Kalman filter, linear quadratic regulator; H_∞ control.

2 Prerequisite

MAE 3260, ECE 3250, or permission of instructor.
Recommended: Good background in linear algebra. Students should be comfortable with mathematical rigor and mathematical proofs. In case of doubt, please contact the instructor.

3 Textbooks

The class will be mainly based on the following (recommended) book:

H18: João P. Hespanha, *Linear Systems Theory: Second Edition*, Princeton University Press, 2018 (pdf available from cornell library)

Additional optional references are:

- John Lygeros and Federico A. Ramponi, *Lectures Notes on Linear Systems Theory*, Jan 2020. (Unpublished, will be made available on canvas)
- Panos J. Antsaklis and Anthony N. Michel, *Linear Systems*, Springer, 2005
- Chi-Tsong Chen, *Linear Systems Theory and Design*, Oxford University Press, 2012
- Wilson J. Rugh, *Linear System Theory*, Pearson, 1996
- Thomas Kailath, *Linear Systems*, Prentice-Hall, 1980

4 Tentative course outline

This is a tentative schedule and may be subject to changes.

Week	Topic	Material
Module 1: Analysis Linear Time Varying Systems (LTV)		
Week 1-2	Introduction; Linearization; Existence and uniqueness for linear time varying systems	H18: Chapters 1,2,5
Module 2: Analysis Linear Time Invariant Systems (LTI)		
Week 3-4	Existence and uniqueness for linear time invariant systems; Impulse response; Transfer function	H18: Chapters 3,4,6,7
Module 3: Stability		
Week 5-7	Lyapunov stability; Schur and Hurwitz matrices; I/O Stability	H18: Chapters 8,9
Module 4: Reachability		
Week 8-9	Reachability; Controllability; Stabilizability	H18: Chapters 11,12,13,14
Module 5: Observability and output feedback		
Week 10-11	Observability; Kalman decomposition; State estimation; Output feedback	H18: Chapters 15,16
Module 6: Realization theory and advanced topics		
Week 12-14	Realization theory; Advanced topics to be decided among LQR, LQG, Kalman filter, H_∞	H18: Chapter 17, lecture notes

5 Teaching modality

This is a 3 credit course, with two lectures per week each of 75 minutes. Lectures will be delivered online in real time. While recordings will be made available, live attendance and participation in class discussions is highly recommended.

6 Assignments and exam:

Homeworks will be assigned approximately every two weeks. For one (and only one) of the homeworks, students will be allowed a late submission to account for unexpected events. If a student chooses this option the corresponding homework should in any case be submitted no more that 3 days late (after which solutions will be uploaded). Homeworks submitted more than three days late will not be evaluated. In any case, the homework with the lowest score will not count towards the final grade. Students will additionally take part in a short project with the objective of applying the theory learned in this class to a system of their own choosing (this may be done in groups depending on total number of students). Finally, there will be a written exam, most likely in take home format given the current situation.

Evaluation:

- Homeworks: 48%
- Final exam: 35%
- Project: 15%
- Participation: 2%